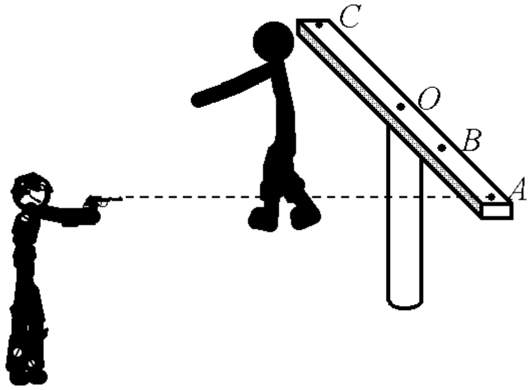
NATIONAL
MATH + SCIENCE
INITIATIVE

AP PHYSICS 1

Rotational Energy and Momentum

Pre-Assessment Questions

James Bond is trying to enter a mansion by sneaking in through the back yard. There, he encounters a guard that is standing in front of a clothesline that consists of a horizontal wooden plank that can pivot and rotate freely on a vertical post at point *O*. In order to take down the guard non-lethally and without making much noise, James Bond shoots a bullet from his silenced Walther PPK pistol at point *A* on the clothesline. The bullet embeds itself in the wood and the wood rotates, causing point *C* to strike the back of the guard's head. The guard is hit just hard enough to knock him unconscious. Answer the following in brief paragraphs that cite specific physical principles.



(a) Explain what would happen if the same bullet was fired at the same speed but at point *B* on the post.

(b) Explain what would happen if the same bullet was fired at the same speed but at point *O* on the post.

(c) Explain what would happen if the same bullet was fired at the same speed but at point *C* on the post.

(d) Explain what would happen if a rubber bullet of the same mass is fired at the same speed at point *A* on the post. Assume the rubber bullet bounces elastically off of the post.

Rotational and Translational Kinetic Energies

Kinetic energy is the energy of motion. Rotation is a type of motion. Therefore, objects that are rotating have kinetic energy. When the center of mass of an object moves from one location to another, this motion is called translational motion. Objects that exhibit translational motion have **translational kinetic energy**.

When the atoms of an object make circles around the center of mass (or a fixed axis), this motion is called rotational motion. Objects that exhibit rotational motion have **rotational kinetic energy**. It is possible for an object to have both translational and rotational kinetic energies at the same time.

Translational Kinetic Energy	Rotational Kinetic Energy
$K_T = \frac{1}{2}mv^2$ m = Mass (difficulty to change translational motion) v = Linear speed (how fast the object translates)	$K_R = \frac{1}{2}I\omega^2$ I = Rotational inertia (difficulty to change rotation) v = Angular speed (how fast the object rotates)

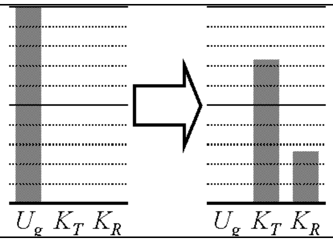
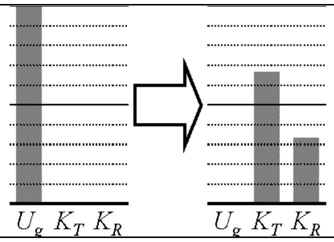
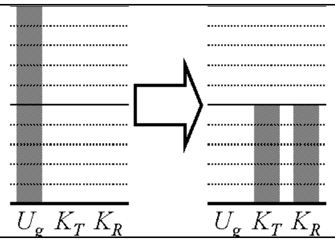
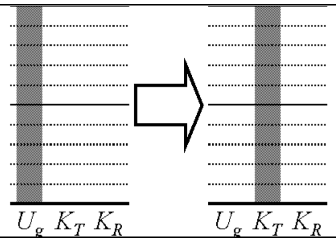
Rolling

An object that is rolling has both translational and rotational kinetic energy. There are two regimes for how an object can roll:

- **Rolling without slipping**—the object rolls along the surface without "peeling out" or "skidding out". Static friction is the frictional force that acts. The static friction force does NO work, which means that friction does not take away mechanical energy.
- **Rolling with slipping**—the object rolls along the surface while either "peeling out" (rotating faster than it is moving) or "skidding out" (rotating slower than it is moving). Kinetic friction acts, and kinetic friction takes away mechanical energy.

If an object rolls down an incline *without slipping* from rest, gravitational potential energy (GPE) becomes the sum of translational and rotational kinetic energies (TKE and RKE). When GPE becomes TKE+RKE, more energy goes to RKE for objects that have higher rotational inertia (have more mass distributed farther from the rotation axis). The more GPE goes to RKE, the less energy goes to TKE, and the slower the object's center of mass moves at the bottom of the incline.

Consider these objects released from rest at the top of an incline: a sphere, a solid disk, and a ring made of some metal material, and a cube of ice. The sphere, disk, and ring have equal masses and radii and do not slip as they roll, but the cube of ice (having the same mass) experiences no friction.

			
The sphere has the least rotational inertia, so the least energy goes to RKE. The sphere moves fast at the bottom of the incline because it has lots of TKE.	The solid disk has more rotational inertia, so more GPE becomes RKE. The disk moves slower than the sphere at the bottom because it has less TKE.	The ring has the most rotational inertia possible, so half of the GPE becomes RKE. Because the least energy goes to TKE, the ring moves the slowest at the bottom.	The ice experiences no friction, so the ice never rotates. Since no GPE becomes RKE and instead becomes all TKE, the ice moves the fastest at the bottom.

If an object rolls down an incline and slips, then pat of the GPE turns into TKE+RKE, but some of the energy is lost to frictional effects.

Angular Momentum

If an object's center of mass translates (moves) from one place to another, the object has linear momentum. If the atoms of the object circle around the center of mass or an axis (rotation), the object has angular momentum.

Misconception	Correct Conceptions
Angular momentum can become linear momentum, or linear momentum can become angular momentum.	Linear momentum and angular momentum are completely separate quantities. • Linear momentum in one object can become linear momentum in another object. • Angular momentum in one object can be transferred and become angular momentum in another object. But transferring angular momentum to linear momentum (or vice versa) makes as much sense as transferring apples to spark plugs.

Angular momentum can only be measured once an origin is chosen. We then say we are measuring "angular momentum about (the origin)". For most situations, the angular momentum should be taken about a rotation axis. *The amount of angular momentum depends on the origin chosen*. Consider Earth's angular momentum:

- If the angular momentum is measured about Earth's axis, then the angular momentum would be the result of the Earth's rotation about its own axis.
- If the angular momentum is measured about the center of the Sun, then the angular momentum would be the result of Earth's orbit around the Sun. This would be a completely different amount of angular momentum!

Angular momentum can be held by objects that (1) rotate about the chosen axis, or (2) move in a direction that is neither directly toward nor directly away from the chosen axis.

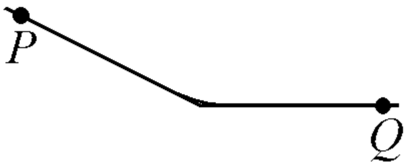
Angular Momentum of a Rotating Object	Angular Momentum of a Translating Object
$L = I\omega$ I = Rotating object's rotational inertia ω = Rotating object's angular speed	$L = mvr\sin\theta$ m = Translating object's mass v = Translating object's speed r = The vector that points from the chosen origin to the object θ = The angle between the v and r vectors

Conservation of Angular Momentum

The angular momentum of an object or system of objects remains constant unless an external force applies a net torque to the system (called "an external torque"). This is **Conservation of Angular Momentum**. If an external torque does act on a system, then it takes time for the torque to add or take away angular momentum. The equation that relates torque applied, time the torque is applied, and gain/loss of angular momentum is $\Delta L = \tau_{net}\Delta t$. This is the **Rotational Impulse-Momentum Theorem**.

An example of this is the Earth-Moon system. As viewed from above the North Pole, Earth rotates counterclockwise, and the Moon orbits counterclockwise. But the Earth is slowing down in its CCW rotation, and the Moon is speeding up in its CCW orbital motion. What is happening is that the Earth is exerting a CCW torque on the Moon (giving the Moon more CCW orbital angular momentum) and the Moon is exerting an equal (but opposite) CW torque on the Earth (taking away Earth's rotational angular momentum). While Earth and Moon's angular momentums individually change, the angular momentum of the Earth-Moon system remains constant.

Multiple-Choice Questions

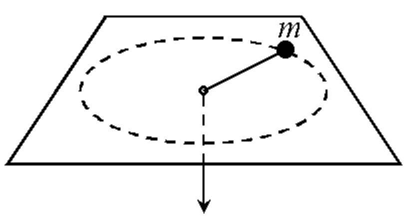


Questions 1–2: The diagram shows an inclined track connected to a flat section of track. Point *P* is near the top of the incline and point *Q* is near the other end of the flat section of track. All of the following objects are released from rest at point *P* and roll without slipping to point *Q*: a solid sphere, a hollow sphere, and a hollow ring. All three objects have the same mass and radius. Use these answer choices for these two questions:

(A) Solid Sphere
(B) Hollow Sphere
(C) Hollow Ring
(D) All three objects have the same upon reaching *Q*.

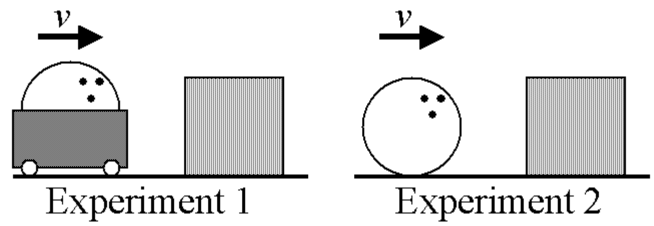
M1. Which object has the greatest total kinetic energy upon reaching *Q*?

M2. Which object has the greatest speed upon reaching *Q*?



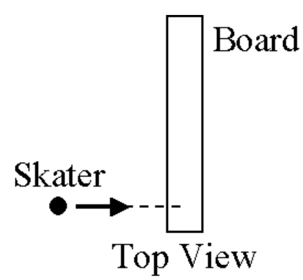
M3. A sphere *m* on the end of a string moves in a circle on a horizontal frictionless table. If the string is pulled through a hole in the table, how do the kinetic energy of the sphere and the sphere's angular momentum about the hole change?

	Kinetic Energy	Angular Momentum
(A)	Increases	Remains Constant
(B)	Decreases	Remains Constant
(C)	Remains Constant	Increases
(D)	Remains Constant	Decreases



M4. In Experiment 1, a student places a bowling ball in a light cart with frictionless bearings. The student sends the cart toward a box with speed *v*. When the cart and box collide, both travel one meter as friction brings them to rest. In Experiment 2, the student removes the ball from the cart and rolls the ball with the same speed *v* toward the same cart. The ball never slips as it rolls, and the student predicts that the box will again slide one meter before coming to rest after the ball collides with it. However, the student observes that the ball pushes the cart 1.4 meters before the box comes to rest. Which of the following best explains why the student made the incorrect prediction?

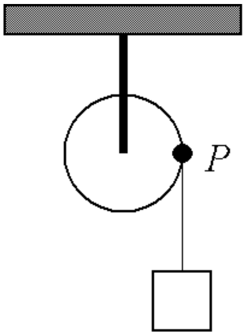
(A) Friction acted on the cart in Experiment 1 but not on the ball in Experiment 2.
(B) The student did not account for all of the ball's kinetic energy in Experiment 2.
(C) The ball-cart in Experiment 1 had more mass than the ball in Experiment 2 had.
(D) The collision was not elastic in either experiment.



M5. A skater on an icy pond skates toward a long board. The skater jumps onto one end of the board and rides on the board. Of linear momentum p, angular momentum L, and kinetic energy K, which quantities are conserved for the skater-board system during the collision?

(A) *L* only
(B) *L* and *p* only
(C) *p* and *K* only
(D) *L*, *p*, and *K*

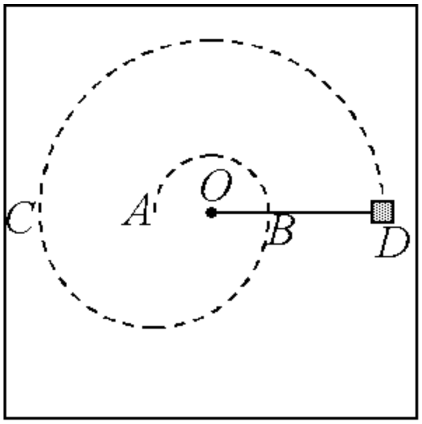
M6. A disk supported by a frictionless axis has several turns of string wrapped around it. The free end of the string is connected to a block as shown in the diagram. When the block is released, the angular momentum of the disk-block system about point *P* changes with time. Which forces are responsible for the changing angular momentum? **Select two answers.**



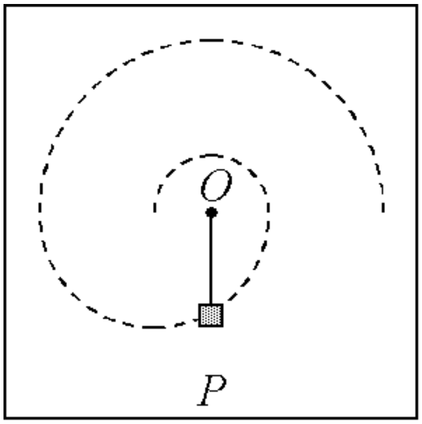
(A) Weight of the block
(B) Weight of the disk
(C) Tension on the disk
(D) Force supporting the disk

Free-Response Questions

F1. The diagram shows a top-down view of a frictionless tabletop that has a small hole in the middle at point *O*. A small block is connected to a string whose other end passes through the hole and is held onto by a person. The block performs uniform circular motion in a small circle clockwise around the hole. The block passes between point *A* and *B* while traveling along this small circular path. As the block moves from *B* to *C*, the person holding the string releases a length of the string so that the block's distance from the hole increases. As the block travels from *C* to *D*, the block is now traveling in a new uniform circular motion around a circle of greater radius.



(a) i. The diagram to the right shows the box at point *P* at an instant during the time that the string is being lengthened. On the box, draw a vector labeled *v* representing the velocity of the box at this instant, and a vector labeled *F* that represents the tension force acting on the box at this instant.



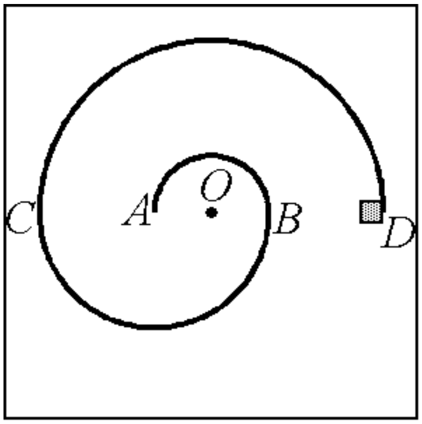
ii. Which of the following is happening as the block travels from point *B* to point *C*?

The kinetic energy: _____ Increases _____ Decreases _____ Remains the same

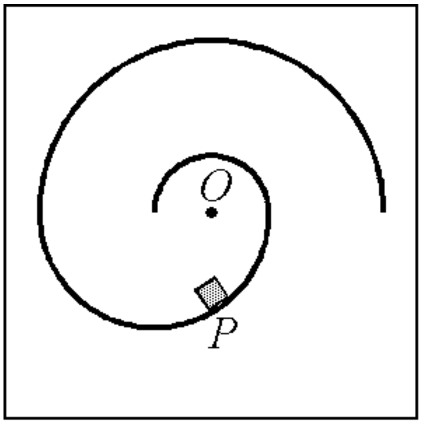
The angular momentum about point *O*: _____ Increases _____ Decreases _____ Remains the same

iii. Justify both of your answers.

Now consider a new frictionless tabletop that has a frictionless wall attached to its surface. Between points *A* and *B*, the wall is a semicircle with a large radius centered on point *O*. Between points *C* and *D*, the wall is a semicircle with a small radius centered on point *O*. A segment of wall connects points *B* and *C* so that the wall gets farther from point *O* as one travels from point B to point *C*. A block is given an initial motion so that the block comes in contact with the wall at point *A* and follows the wall all the way around to point *D*.



(b) i. The diagram to the right shows the box at point *P* at an instant during the time that the box is moving farther away from point *O*. On the box, draw a vector labeled *v* representing the velocity of the box at this instant, and a vector labeled *F* that represents the normal force acting on the box at this instant.



ii. Which of the following is happening as the block travels from point *B* to point *C*?

The kinetic energy: _____Increases _____ Decreases _____ Remains the same

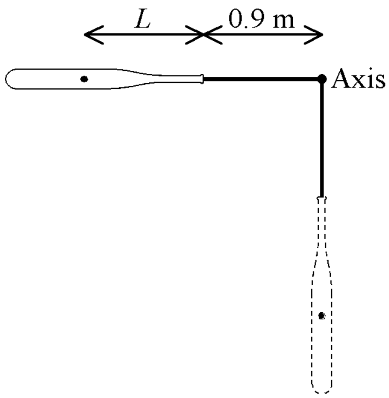
The angular momentum about point *O*: _____Increases _____ Decreases _____ Remains the same

iii. Justify both of your answers.

F2. A student studying baseball wishes to determine the rotational inertia of a baseball bat as it is rotated about an axis that passes through a batter as the batter swings to hit a baseball.

(a) Briefly explain how the student can determine the location of the bat's center of mass.

The student's research indicates that the batter holds the handle of the bat a distance of 0.9 m from the axis of rotation. The student's experiment to determine the rotational inertia of the bat consists of the set-up shown. A light rod that is 0.9 m long is glued to the handle of the bat, and the other end of the rod is connected to a frictionless axis. The bat-and-rod assembly is held horizontal as shown in the diagram and released from rest. The assembly is allowed to rotate through the vertical position as shown. In the diagram, the center of mass is indicated by the dot. Let L represent the distance from the handle end to the bat's center of mass that was determined in part (a).



(b) What measurements must the student make in order to calculate the rotational inertia of the bat about the axis? Explain how, and with what equipment, the student can make those measurements.

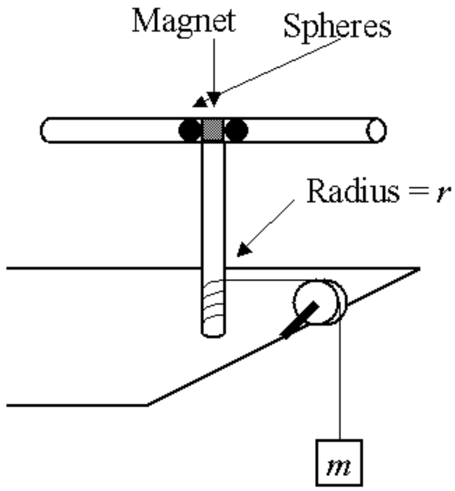
(c) Explain how the measurements that the student made in part (b) can be used to calculate the rotational inertia of the bat about the axis. Show specific equations and explain how they will be used.

Now consider a batter swinging the bat and striking a pitched baseball. Just before the bat strikes the ball, the ball is moving horizontally in a direction perpendicular to the bat's length. Upon striking the bat, the baseball completely reverses direction and has a greater speed than before being struck. Consider the system consisting of the batter, bat, and baseball.

(d) Is the angular momentum of this system the same after the ball is struck as before the ball was struck? Explain your reasoning.

(e) Is the linear momentum of this system the same after the ball is struck as before the ball was struck? Explain your reasoning.

F3. Students are investigating the relationship between the torque applied to a rotating object and its subsequent motion. The students build the apparatus shown, which consists of a vertical pipe of radius r connected to a horizontal pipe at the top. An electromagnet inside the top pipe holds two heavy spheres at the center of rotation. The vertical pipe can rotate on an axis fixed to a table with no friction. Students wrap several turns of string around the vertical pipe and pass the string over an ideal pulley so that the free end of the string connects to a hanging mass m.



(a) When the system is released, the mass falls with constant acceleration. The tension in the string is T and the rotational inertia of the apparatus is I. Derive expressions for the following as functions of time t in terms of I, T, and the radius r of the vertical pipe.

i. The angular momentum L of the rotating apparatus

ii. The kinetic energy K of the rotating apparatus

iii. The speed at which the mass m falls

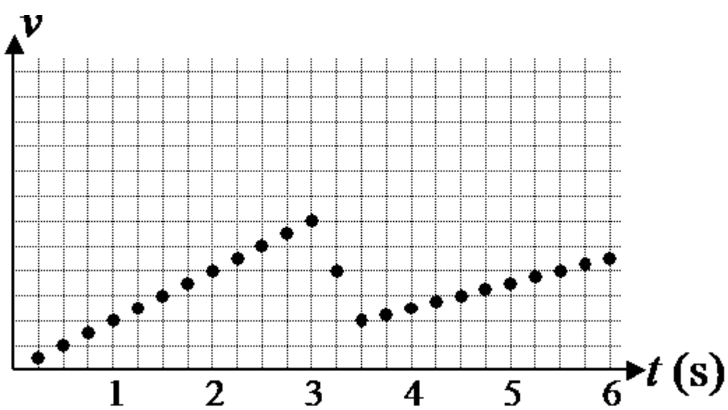
(b) Suppose that the experiment is repeated but this time the magnet is turned off so that the spheres move to the ends of the horizontal pipe where they remain. The students note that the mass m falls slower than in the first trial. They are attempting to explain why this is. Their arguments are as follows:

Student 1: "When the spheres move to the ends, the rotational inertia increases. The tension is the same because the tension is a constant force since it results in a constant acceleration in the hanging mass. If the tension is the same, the torque is the same, and if the torque is the same and the rotational inertia is greater, then there is less rotation. If there is less rotation, the string unwinds less and that means that the block moves less."

Student 2: "I don't think that the spheres count as part of the rotational inertia of the apparatus since the spheres don't ever roll—they just stay in the same spot on the apparatus in both cases. If the rotational inertia is the same, then there must be less tension providing less torque. This also makes sense because less tension acting on the hanging mass results in less acceleration of the hanging mass according to $F = ma$."

Write a clear paragraph that synthesizes the correct aspects of the students' statements into a single coherent explanation for why the hanging mass moves slower in the second experiment. A complete solution would address how quantities cited by the students change and discuss relevant physical principles.

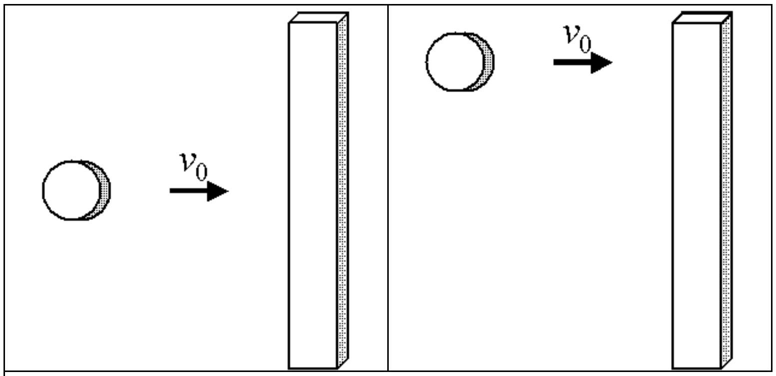
(c) In a third experiment, the magnet is initially on and the spheres are in the center. At time $t = 0$, the apparatus and hanging mass are released from rest. At time $t = 3$ seconds, the magnet is turned off and the spheres move to the ends of the horizontal pipe and the apparatus is allowed to continue rotating. The downward speed v of the mass is shown in the graph as a function of time t.



i. Explain why the graph has a greater slope during the interval 0 to 3 seconds than during the interval 4 to 6 seconds.

ii. Explain why the graph decreases in value just after time $t = 3$ seconds.

F4. Somewhere in space far from the gravitational effects of any large objects, an astronaut performs an experiment in which a magnetic puck approaches an iron bar. In each case, the puck initially has a constant velocity v_0 that is directed perpendicular to the length of the bar. Upon colliding, the puck sticks to that point on the bar that the puck collided with. In Case 1, the puck approaches and strikes the center of the bar, and in Case 2, the puck approaches and strikes one end of the bar.



Case 1　　Case 2

(a) i. In which case(s) is the linear momentum of the system conserved as a result of the collision?

___ Neither case　___ Case 1 only　___ Case 2 only　___ Both cases

ii. In which case(s) is the angular momentum about the system center of mass conserved as a result of the collision?

___ Neither case　___ Case 1 only　___ Case 2 only　___ Both cases

iii. Explain both of your answers.

(b) In which case does a greater amount of kinetic energy transform into other forms of energy as a result of the collision?

___ Case 1　___ Case 2　___ Both cases transform the same amount of energy

In a well-organized, paragraph-length response that may include equations and/or diagrams, explain your reasoning. Be sure to cite appropriate physical principles.