NAME ______________________ DATE ______________ CLASS ______________

Holt Physics

Problem 3A

FINDING RESULTANT MAGNITUDE AND DIRECTION

PROBLEM

Cheetahs are, for short distances, the fastest land animals. In the course of a chase, cheetahs can also change direction very quickly. Suppose a cheetah runs straight north for 5.0 s, quickly turns, and runs 3.00×10^2 m west. If the magnitude of the cheetah's resultant displacement is 3.35×10^2 m, what is the cheetah's displacement and velocity during the first part of its run?

SOLUTION

1. DEFINE **Given:**

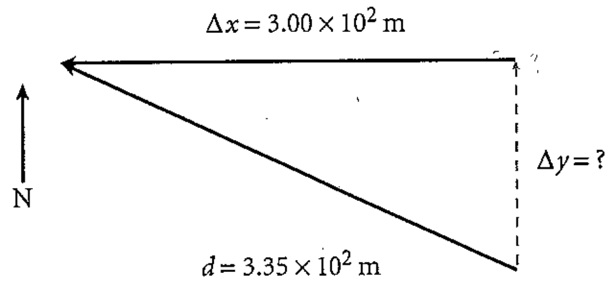
$\Delta t_1 = 5.0$ s

$\Delta x = 3.00 \text{ s} \times 10^2$ m

$d = 3.35 \times 10^2$ m

Unknown: $\Delta y = ?$ $\quad v_y = ?$

Diagram:



2. PLAN **Choose the equation(s) or situation:** Use the Pythagorean theorem to subtract one of the displacements at right angles from the total displacement, and thus determine the unknown component of displacement.

$$d^2 = \Delta x^2 + \Delta y^2$$

Use the equation relating displacement to constant velocity and time, and use the calculated value for Δy and the given value for Δt to solve for v.

$$\Delta v = \frac{\Delta y}{\Delta t}$$

Rearrange the equation(s) to isolate the unknown(s):

$$\Delta y^2 = d^2 - \Delta x^2$$

$$\Delta y = \sqrt{d^2 - \Delta x^2}$$

$$v_y = \frac{\Delta y}{\Delta t}$$

3. CALCULATE **Substitute the values into the equation(s) and solve:** Because the value for Δy is a displacement magnitude, only the positive root is used ($\Delta y > 0$).

$$\Delta y = \sqrt{(3.35 \times 10^2 \text{ m})^2 - (3.00 \times 10^2 \text{ m})^2}$$

$$= \sqrt{1.12 \times 10^5 \text{ m}^2 - 9.00 \times 10^4 \text{ m}^2}$$

$$= \sqrt{2.2 \times 10^4\ \text{m}}$$

$$= \boxed{1.5 \times 10^2\ \text{m, north}}$$

$$v_y = \frac{1.5 \times 10^2\ \text{m}}{5.0\ \text{s}} = \boxed{3.0 \times 10^1\ \text{m/s, north}}$$

4. EVALUATE The cheetah has a top speed of 30 m/s, or 107 km/h. This is equal to about 67 miles/h.

ADDITIONAL PRACTICE

1. An ostrich cannot fly, but it is able to run fast. Suppose an ostrich runs east for 7.95 s and then runs 161 m south, so that the magnitude of the ostrich's resultant displacement is 226 m. Calculate the magnitude of the ostrich's eastward component and its running speed.
2. The pronghorn antelope, found in North America, is the best long-distance runner among mammals. It has been observed to travel at an average speed of more than 55 km/h over a distance of 6.0 km. Suppose the antelope runs a distance of 5.0 km in a direction 11.5° north of east, turns, and then runs 1.0 km south. Calculate the resultant displacement.
3. Kangaroos can easily jump as far 8.0 m. If a kangaroo makes five such jumps westward, how many jumps must it make northward to have a northwest displacement with a magnitude of 68 m? What is the angle of the resultant displacement with respect to north?
4. In 1926, Gertrude Ederle of the United States became the first woman to swim across the English channel. Suppose Ederle swam 25.2 km east from the coast near Dover, England, then made a 90° turn and traveled south for 21.3 km to a point east of Calais, France. What was Ederle's resultant displacement?
5. The emperor penguin is the best diver among birds: the record dive is 483 m. Suppose an emperor penguin dives vertically to a depth of 483 m and then swims horizontally a distance of 225 m. What angle would the vector of the resultant displacement make with the water's surface? What is the magnitude of the penguin's resultant displacement?
6. A killer whale can swim as fast as 15 m/s. Suppose a killer whale swims in one direction at this speed for 8.0 s, makes a 90° turn, and continues swimming in the new direction with the same speed as before. After a certain time interval, the magnitude of the resultant displacement is 180.0 m. Calculate the amount of time the whale swims after changing direction.
7. Woodcocks are the slowest birds: their average speed during courtship displays can be as low as 8.00 km/h. Suppose a woodcock flies east for 15.0 min. It then turns and flies north for 22.0 min. Calculate the magnitude of the resultant displacement and the angle between the resultant displacement and the woodcock's initial displacement.

NAME ______________________ DATE ____________ CLASS ____________

Holt Physics

Problem 3B

RESOLVING VECTORS

PROBLEM

Certain iguanas have been observed to run as fast as 10.0 m/s. Suppose an iguana runs in a straight line at this speed for 5.00 s. The direction of motion makes an angle of 30.0° to the east of north. Find the value of the iguana's northward displacement.

SOLUTION

1. DEFINE **Given:**

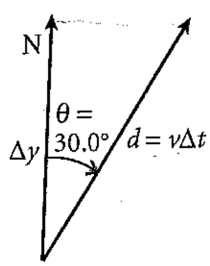
$v = 10.0$ m/s

$t = 5.00$ s

$\theta = 30.0°$

Unknown: $\Delta y = ?$

Diagram:



2. PLAN **Choose the equation(s) or situation:** The northern component of the vector is equal to the vector magnitude times the cosine of the angle between the vector and the northward direction.

$$\Delta y = d(\cos \theta)$$

Use the equation relating displacement with constant velocity and time, and substitute it for d in the previous equation.

$$d = v\Delta t$$
$$\Delta y = v\Delta t(\cos \theta)$$

3. CALCULATE **Substitute the values into the equation(s) and solve:**

$$\Delta y = \left(10.0\frac{\text{m}}{\text{s}}\right)(5.00 \text{ s})(\cos 30.0°)$$

$$= \boxed{43.3 \text{ m, north}}$$

4. EVALUATE The northern component of the displacement vector is smaller than the displacement itself, as expected.

ADDITIONAL PRACTICE

1. A common flea can jump a distance of 33 cm. Suppose a flea makes five jumps of this length in the northwest direction. If the flea's northward displacement is 88 cm, what is the flea's westward displacement.

2. The longest snake ever found was a python that was 10.0 m long. Suppose a coordinate system large enough to measure the python's length is drawn on the ground. The snake's tail is then placed at the origin and the snake's body is stretched so that it makes an angle of 60.0° with the positive x-axis. Find the x and y coordinates of the snake's head. (Hint: The y-coordinate is positive.)

3. A South-African sharp-nosed frog set a record for a triple jump by traveling a distance of 10.3 m. Suppose the frog starts from the origin of a coordinate system and lands at a point whose coordinate on the y-axis is equal to –6.10 m. What angle does the vector of displacement make with the negative y-axis? Calculate the x component of the frog.

4. The largest variety of grasshopper in the world is found in Malaysia. These grasshoppers can measure almost a foot (0.305 m) in length and can jump 4.5 m. Suppose one of these grasshoppers starts at the origin of a coordinate system and makes exactly eight jumps in a straight line that makes an angle of 35° with the positive x-axis. Find the grasshopper's displacements along the x- and y-axes. Assume both component displacements to be positive.

5. The landing speed of the space shuttle *Columbia* is 347 km/h. If the shuttle is landing at an angle of 15.0° with respect to the horizontal, what are the horizontal and the vertical components of its velocity?

6. In Virginia during 1994 Elmer Trett reached a speed of 372 km/h on his motorcycle. Suppose Trett rode northwest at this speed for 8.7 s. If the angle between east and the direction of Trett's ride was 60.0°, what was Trett's displacement east? What was his displacement north?

7. The longest delivery flight ever made by a twin-engine commercial jet took place in 1990. The plane covered a total distance of 14 890 km from Seattle, Washington to Nairobi, Kenya in 18.5 h. Assuming that the plane flew in a straight line between the two cities, find the magnitude of the average velocity of the plane. Also, find the eastward and southward components of the average velocity if the direction of the plane's flight was at an angle of 25.0° south of east.

8. The French bomber *Mirage IV* can fly over 2.3×10^3 km/h. Suppose this plane accelerates at a rate that allows it to increase its speed from 6.0×10^2 km/h to 2.3×10^3 km/h. in a time interval of 120 s. If this acceleration is upward and at an angle of 35° with the horizontal, find the acceleration's horizontal and vertical components.

Holt Physics

Problem 3C

ADDING VECTORS ALGEBRAICALLY

PROBLEM

The record for the longest nonstop closed-circuit flight by a model airplane was set in Italy in 1986. The plane flew a total distance of 1239 km. Assume that at some point the plane traveled 1.25×10^3 m to the east, then 1.25×10^3 m to the north, and finally 1.00×10^3 m to the southeast. Calculate the total displacement for this portion of the flight.

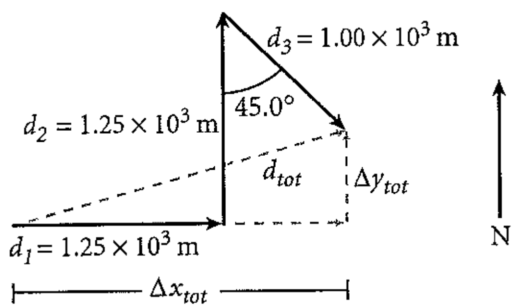
SOLUTION

1. DEFINE

Given: $d_1 = 1.25 \times 10^3$ m $d_2 = 1.25 \times 10^3$ m $d_3 = 1.00 \times 10^3$ m

Unknown: $\Delta x_{tot} = ?$ $\Delta y_{tot} = ?$ $d = ?$ $\theta = ?$

Diagram:



2. PLAN

Choose the equation(s) or situation: Orient the displacements with respect to the x-axis of the coordinate system.

$$\theta_1 = 0.00° \qquad \theta_2 = 90.0° \qquad \theta_3 = -45.0°$$

Use this information to calculate the components of the total displacement along the x-axis and the y-axis.

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 + \Delta x_3$$
$$= d_1(\cos\theta_1) + d_2(\cos\theta_2) + d_3(\cos\theta_3)$$
$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 + \Delta y_3$$
$$= d_1(\sin\theta_1) + d_2(\sin\theta_2) + d_3(\sin\theta_3)$$

Use the components of the total displacement, the Pythagorean theorem, and the tangent function to calculate the total displacement.

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} \qquad \theta = tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right)$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\Delta x_{tot} = (1.25 \times 10^3 \text{ m})(\cos 0°) + (1.25 \times 10^3 \text{ m})(\cos 90.0°)$$
$$+ (1.00 \times 10^3 \text{ m})[\cos(-45.0°)]$$
$$= 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m}$$
$$= 1.96 \times 10^3 \text{ m}$$
$$\Delta y_{tot} = (1.25 \times 10^3 \text{ m})(\sin 0°) + (1.25 \times 10^3 \text{ m})(\sin 90.0°)$$
$$+ (1.00 \times 10^3 \text{ m})[\sin(-45.0°)]$$
$$= 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m}$$
$$= 0.543 \times 10^3 \text{ m}$$

$$d = \sqrt{(1.96 \times 10^3 \text{ m})^2 + (0.543 \times 10^3 \text{ m})^2}$$

$$d = \sqrt{3.84 \times 10^6\ m^2 + 2.95 \times 10^5\ m^2} = \sqrt{4.14 \times 10^6\ m^2}$$

$$d = \boxed{2.03 \times 10^3\ m}$$

$$\theta = tan^{-1}\left(\frac{0.543 \times 10^3\ m}{1.96 \times 10^3\ m}\right)$$

$$\theta = \boxed{15.5° \text{ north of east}}$$

4. EVALUATE The magnitude of the total displacement is slightly larger than that of the total displacement in the eastern direction alone.

ADDITIONAL PRACTICE

1. For six weeks in 1992, Akira Matsushima, from Japan, rode a unicycle more than 3000 mi across the United States. Suppose Matsushima is riding through a city. If he travels 250.0 m east on one street, then turns counterclockwise through a 120.0° angle and proceeds 125.0 m northwest along a diagonal street, what is his resultant displacement?

2. In 1976, the Lockheed SR-71A *Blackbird* set the record speed for any airplane: 3.53×10^3 km/h. Suppose you observe this plane ascending at this speed. For 20.0 s, it flies at an angle of 15.0° above the horizontal, then for another 10.0 s its angle of ascent is increased to 35.0°. Calculate the plane's total gain in altitude, its total horizontal displacement, and its resultant displacement.

3. Magnor Mydland of Norway constructed a motorcycle with a wheelbase of about 12 cm. The tiny vehicle could be ridden at a maximum speed 11.6 km/h. Suppose this motorcycle travels in the directions d_1 and d_2 shown in the figure below. Calculate d_1 and d_2, and determine how long it takes the motorcycle to reach a net displacement of 2.0×10^2 m to the right?

4. The fastest propeller-driven aircraft is the Russian TU-95/142, which can reach a maximum speed of 925 km/h. For this speed, calculate the plane's resultant displacement if it travels east for 1.50 h, then turns 135° northwest and travels for 2.00 h.

5. In 1952, the ocean liner *United States* crossed the Atlantic Ocean in less than four days, setting the world record for commercial ocean-going vessels. The average speed for the trip was 57.2 km/h. Suppose the ship moves in a straight line eastward at this speed for 2.50 h. Then, due to a strong local current, the ship's course begins to deviate northward by 30.0°, and the ship follows the new course at the same speed for another 1.50 h. Find the resultant displacement for the 4.00 h period.

NAME ______________________ DATE ____________ CLASS ______________

Holt Physics

Problem 3E

PROJECTILES LAUNCHED AT AN ANGLE

PROBLEM

The narrowest strait on earth is Seil Sound in Scotland, which lies between the mainland and the island of Seil. The strait is only about 6.0 m wide. Suppose an athlete wanting to jump "over the sea" leaps at an angle of 35° with respect to the horizontal. What is the minimum initial speed that would allow the athlete to clear the gap? Neglect air resistance.

SOLUTION

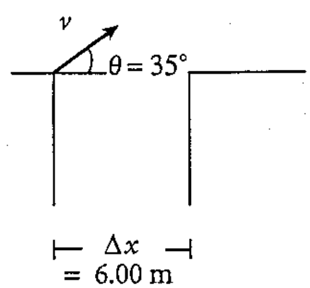
1. DEFINE **Given:** $\Delta x = 6.0$ m

$\theta = 35°$

$g = 9.81$ m/s

Unknown: $v_i = ?$

2. PLAN **Diagram:**



Choose the equation(s) or situation: The horizontal component of the athlete's velocity, v_x, is equal to the initial speed multiplied by the cosine of the angle, θ, which is equal to the magnitude of the horizontal displacement, Δx, divided by the time interval required for the complete jump.

$$v_x = v_i \cos\theta = \frac{\Delta x}{\Delta t}$$

At the midpoint of the jump, the vertical component of the athlete's velocity, v_y, which is the upward vertical component of the initial velocity, $v_i \sin\theta$, minus the downward component of velocity due to free-fall acceleration, equals zero. The time required for this to occur is half the time necessary for the total jump.

$$v_y = v_i \sin\theta - g\left(\frac{\Delta t}{2}\right) = 0$$

$$v_i \sin\theta = \frac{g\Delta t}{2}$$

Rearrange the equation(s) to isolate the unknown(s): Express Δt in the second equation in terms of the displacement and velocity component in the first equation.

$$v_i \sin\theta = \frac{g}{2}\left(\frac{\Delta x}{v_i \cos\theta}\right)$$

$$v_i^2 = \frac{g\Delta x}{2\sin\theta\cos\theta}$$

$$v_i = \sqrt{\frac{g\Delta x}{2 \sin \theta \cos \theta}}$$

3. CALCULATE **Substitute the values into the equation(s) and solve:** Select the positive root for v_i.

$$v_i = \sqrt{\frac{\left(9.81 \frac{m}{s^2}\right)(6.0 \text{ m})}{(2)(\sin 35°)(\cos 35°)}} = \boxed{7.9 \frac{m}{s}}$$

4. EVALUATE By substituting the value for v_i into the original equations, you can determine the time for the jump to be completed, which is 0.92 s. From this, the height of the jump is found to equal 1.0 m.

ADDITIONAL PRACTICE

1. In 1993, Wayne Brian threw a spear a record distance of 201.24 m. (This is not an official sport record because a special device was used to "elongate" Brian's hand.) Suppose Brian threw the spear at a 35.0° angle with respect to the horizontal. What was the initial speed of the spear?
2. April Moon set a record in flight shooting (a variety of long-distance archery). In 1981 in Utah, she sent an arrow a horizontal distance of 9.50×10^2 m. What was the speed of the arrow at the top of the flight if the arrow was launched at an angle of 45.0° with respect to the horizontal?
3. In 1989 during overtime in a high school basketball game in Erie, Pennsylvania, Chris Eddy threw a basketball a distance of 27.5 m to score and win the game. If the shot was made at a 50.0° angle above the horizontal, what was the initial speed of the ball?
4. In 1978, Geoff Capes of the United Kingdom won a competition for throwing 5 lb bricks; he threw one brick a distance of 44.0 m. Suppose the brick left Capes' hand at an angle of 45.0° with respect to the horizontal.
   - **a.** What was the initial speed of the brick?
   - **b.** What was the maximum height reached by the brick?
   - **c.** If Capes threw the brick straight up with the speed found in (a), what would be the maximum height the brick could achieve?
5. In 1991, Doug Danger rode a motorcycle to jump a horizontal distance of 76.5 m. Find the maximum height of the jump if his angle with respect to the ground at the beginning of the jump was 12.0°.
6. Michael Hout of Ohio can run 110.0 meter hurdles in 18.9 s at an average speed of 5.82 m/s. What makes this interesting is that he juggles three balls as he runs the distance. Suppose Hout throws a ball up and forward at twice his running speed and just catches it at the same level. At what angle, θ, must the ball be thrown? (Hint: Consider horizontal displacements for Hout and the ball.)

7. A scared kangaroo once cleared a fence by jumping with a speed of 8.42 m/s at an angle of 55.2° with respect to the ground. If the jump lasted 1.40 s, how high was the fence? What was the kangaroo's horizontal displacement?

8. Measurements made in 1910 indicate that the common flea is an impressive jumper, given its size. Assume that a flea's initial speed is 2.2 m/s, and that it leaps at an angle of 21° with respect to the horizontal. If the jump lasts 0.16 s, what is the magnitude of the flea's horizontal displacement? How high does the flea jump?

NAME ______________________ DATE ____________ CLASS ____________

Holt Physics

Problem 3F

RELATIVE VELOCITY

PROBLEM

The world's fastest current is in Slingsby Channel, Canada, where the speed of the water reaches 30.0 km/h. Suppose a motorboat crosses the channel perpendicular to the bank at a speed of 18.0 km/h relative to the bank. Find the velocity of the motorboat relative to the water.

SOLUTION

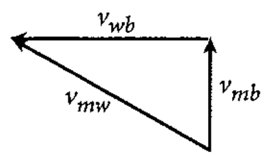
1. DEFINE

Given: $\mathbf{v_{wb}}$ = 30.0 km/h along the channel (velocity of the *water, w,* with respect to the *bank, b)*

$\mathbf{v_{mb}}$ = 18.0 km/h perpendicular to the channel (velocity of the *motorboat, m,* with respect to the *bank, b)*

Unknown: $\mathbf{v_{mw}}$ = ?

Diagram:



2. PLAN

Choose the equation(s) or situation: From the vector diagram, the resultant vector (the velocity of the motorboat with respect to the bank, $\mathbf{v_{mb}}$) is equal to the vector sum of the other two vectors, one of which is the unknown.

$$\mathbf{v_{mw}} = \mathbf{v_{mb}} + \mathbf{v_{wb}}$$

Use the Pythagorean theorem to calculate the magnitude of the resultant velocity, and use the tangent function to find the direction. Note that because the vectors $\mathbf{v_{mb}}$ and $\mathbf{v_{wb}}$ are perpendicular to each other, the product that results from multiplying one by the other is zero. The tangent of the angle between $\mathbf{v_{mb}}$ and $\mathbf{v_{mw}}$ is equal to the ratio of the magnitude of $\mathbf{v_{wb}}$ to the magnitude of $\mathbf{v_{mb}}$.

$$v_{mw}^2 = v_{mb}^2 + v_{wb}^2$$

$$\tan\theta = \frac{v_{wb}}{v_{mb}}$$

Rearrange the equation(s) to isolate the unknown(s):

$$v_{mw} = \sqrt{v_{mb}^2 + v_{wb}^2}$$

$$\theta = \tan^{-1}\left(\frac{v_{wb}}{v_{mb}}\right)$$

3. CALCULATE

Substitute the values into the equation(s) and solve: Choose the positive root for v_{mw}.

$$v_{mw} = \sqrt{\left(18.0\frac{\text{km}}{\text{h}}\right)^2 + \left(30.0\frac{\text{km}}{\text{h}}\right)^2} = \boxed{35.0\frac{\text{km}}{\text{h}}}$$

The angle between v_{mb} and v_{mw} is as follows:

$$\theta = \tan^{-1}\left(\frac{30.0\,\frac{\text{km}}{\text{h}}}{18.0\,\frac{\text{km}}{\text{h}}}\right) = \boxed{\text{59.0° away from the oncoming current}}$$

4. EVALUATE The motorboat must move in a direction 59° with respect to $\mathbf{v_{mb}}$ and against the current, and with a speed of 35.0 km/h in order to move 18.0 km/h perpendicular to the bank.

ADDITIONAL PRACTICE

1. In 1933, a storm occurring in the Pacific Ocean moved with speeds reaching a maximum of 126 km/h. Suppose a storm is moving north at this speed. If a gull flies east through the storm with a speed of 40.0 km/h relative to the air, what is the velocity of the gull relative to Earth?

2. George V. Coast in Antarctica is the windiest place on Earth. Wind speeds there can reach 3.00×10^2 km/h. If a research plane flies against the wind with a speed of 4.50×10^2 km/h relative to the wind, how long does it take the plane to fly between two research stations that are 250 km apart?

3. Turtles are fairly slow on the ground, but they are very good swimmers, as indicated by the reported speed of 9.0 m/s for the leatherback turtle. Suppose a leatherback turtle swims across a river at 9.0 m/s relative to the water. If the current in the river is 3.0 m/s and it moves at a right angle to the turtle's motion, what is the turtle's displacement with respect to the river's bank after 1.0 min?

4. California sea lions can swim as fast as 40.0 km/h. Suppose a sea lion begins to chase a fish at this speed when the fish is 60.0 m away. The fish, of course, does not wait, and swims away at a speed 16.0 km/h. How long would it take the sea lion to catch the fish?

5. The spur-wing goose is one of the fastest birds in the world when it comes to *level* flying: it can reach a speed of 90.0 km/h. Suppose two spur-wing geese are separated by an unknown distance and start flying toward each other at their maximum speeds. The geese pass each other 40.0 s later. Calculate the initial distance between the geese.

6. The fastest snake on Earth is the black mamba, which can move over a short distance at 18.0 km/h. Suppose a mamba moves at this speed toward a rat sitting 12.0 m away. The rat immediately begins to run away at 33.3 percent of the mamba's speed. If the rat jumps into a hole just before the mamba can catch it, determine the length of time that the chase lasts.