NAME ______________________ **DATE** Key

Scenario

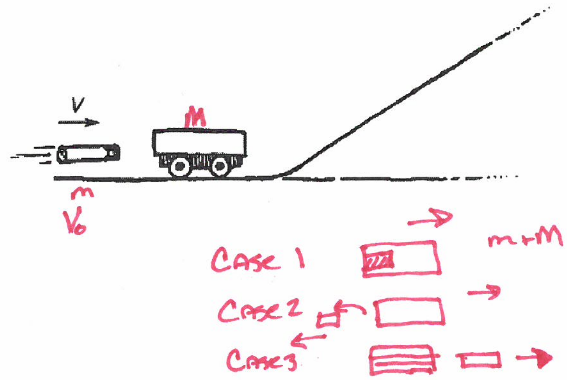
A wooden cart of mass M is set on a horizontal section of track. The cart experiences negligible friction as it rolls. A dart of mass $m < M$ is fired with initial speed v toward the cart, which is initially at rest.

Consider the following cases:

Case 1: The dart embeds itself in the cart.

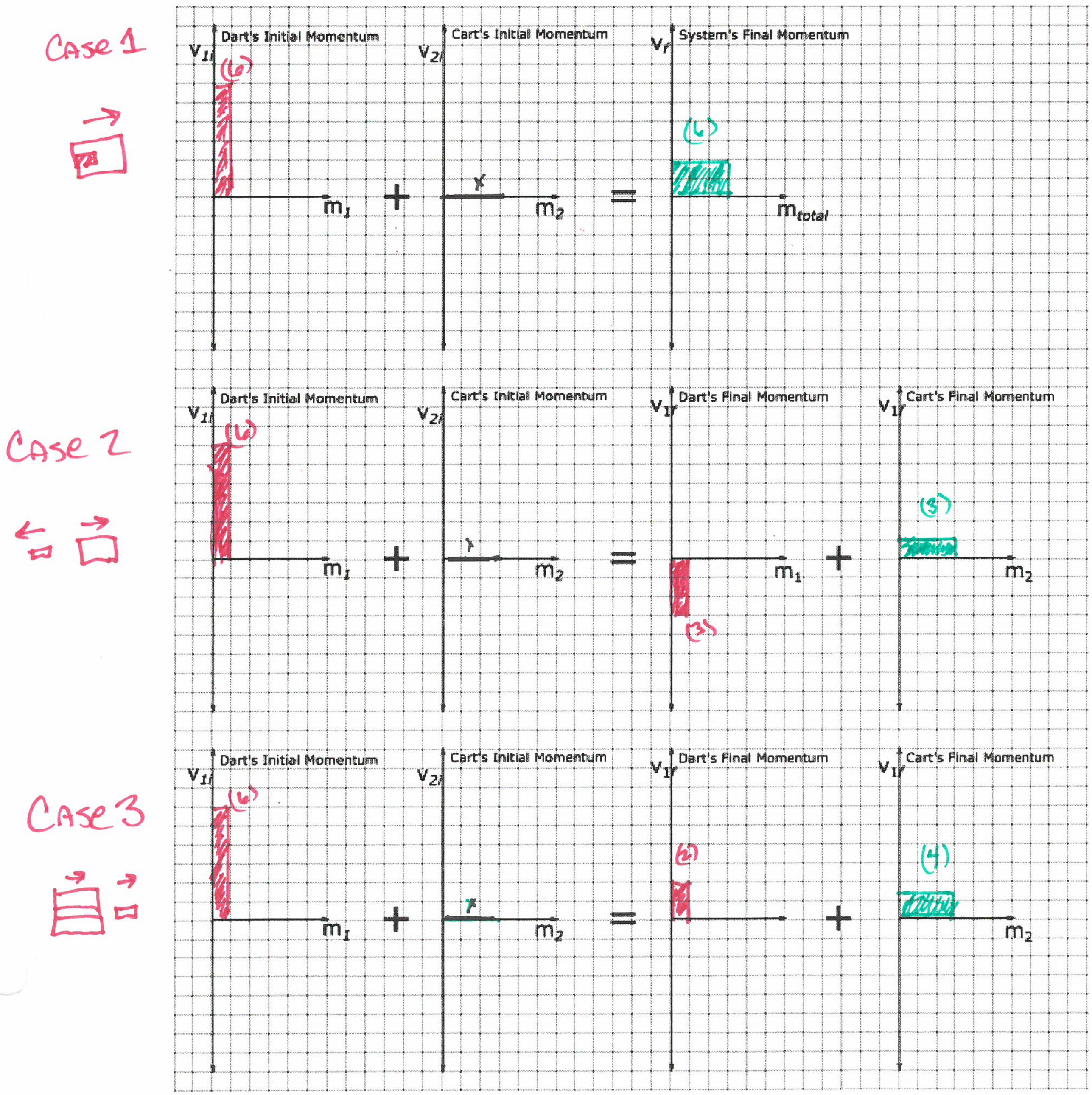
Case 2: The dart bounces backward off the cart.

Case 3: The dart passes through the cart.



Using Representations

PART A: For each case, create a pictorial representation of conservation of momentum during the collision.



Quantitative Analysis

PART B: Create a mathematical representation showing conservation of momentum for each of the three cases. Use each representation to derive the velocity v_{2f} of the cart after the collision.

$V_f = ?$

Case 1	*Case 2*	*Case 3*
$P_i = P_f$ $m_D V_{i_D} + \cancel{m_c V_i} = V_f (m_D + m_c)$ $V_f = \frac{m_D V_{iD}}{m_D + m_c}$	$P_i = P_f$ $m_D V_i + \cancel{m_c V_i} = m_D V_{f_D} + m_c V_{fc}$ $V_f = \frac{m_D V_i + m_D V_f}{m_c}$	$P_i = P_f$ $m_D V_i + \cancel{m_c V_i} = m_D V_{f_D} + m_c V_{fc}$ $V_{fc} = \frac{m_D V_i - m_D V_f}{m_c}$

Argumentation

PART C: Rank the cases in terms of the distance up the incline that the cart travels after its interaction with the dart and explain your ranking in terms of conservation of momentum and conservation of energy.

Claim: Farthest up the incline 2, 1, 3 Least far up the incline

Evidence/Reasoning: (Use evidence and reasoning from Parts A and B to support your claim.)

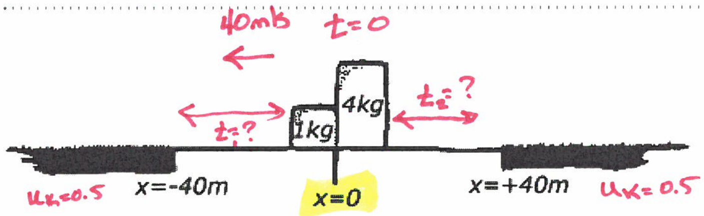
The larger the Velocity of the Cart post-collision the more KE the cart will have, ∴ higher up Ramp

looking at 3 eqns above case 2 produces largest V_f $\left(\frac{\quad}{m_c}\right)$ Then case 1, Then case 3

NAME ____________________ DATE Key

Scenario

Two blocks of mass 1 kg and 4 kg are placed in contact at the center point of an 80 m long track that exerts negligible friction on the blocks. At both ends of the track, there are strips of rough surface of equal coefficient of kinetic friction 0.5. A small explosive charge between the blocks propels them apart at time t = 0 so that the 1 kg block moves with a speed of 40 m/s along its section of the frictionless track.



(This figure is not to scale; the size of the two blocks is significantly smaller than the distance to the location where the frictionless track is connected to a rough surface.)

Quantitative Analysis

PART A: Calculate the time at which the 1 kg block reaches its rough surface and the time at which the 4 kg block reaches its rough surface. Verbally explain your calculations.

1-kg Block	4-kg Block

$P_i = P_f$ During Explosion, No net external forces

$\cancel{m_1 v_i} + \cancel{m_2 v_i} = -m_1 v_{f_1} + m_2 v_{f_2}$ Momentum Conserved ∴ $p_i = p_f$ ← initial momentum ∅

$$v_{f_2} = \frac{m_1 v_{f1}}{m_2} = \frac{(1\,kg)(40\,m/s)}{4\,kg}$$

$$\boxed{v_{f2} = 10\,m/s}$$

Block 1

Frictionless, ∴ acceleration constant $V = \frac{d}{t}$

$D = -40m$

$V_{f1} = -40 m/s$

$t = ?$

$$V = \frac{d}{t}$$

$$t = \frac{d}{V} = \frac{-40m}{-40m/s}$$

$$\boxed{t_1 = 1s}$$

Block 2

$D = 40\,m$

$V = 10 m/s$

$t = ?$

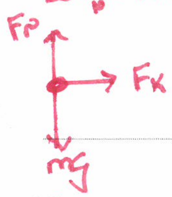
$$t = \frac{d}{V} = \frac{40m}{10 m/s}$$

$$\boxed{t_2 = 4s}$$

1-kg Block	4-kg Block

PART B: Both blocks have the same magnitude of acceleration while sliding to rest on their respective rough surfaces. Calculate this acceleration and verbally explain your method.

$a = ?$

$\Sigma F_x = ma$	The net external force is equal to mass times acceleration
	The only net force is Force of Kinetic Friction
$F_k = ma$ $F_k = F_n \mu_k$ $F_n = mg$ $F_k = \mu_k mg$	Force of Friction equal to Normal force x Coeff Friction
$\mu_k \cancel{m} g = \cancel{m} a$	masses cancal
$a = \mu_k g$ $= (0.5)(9.8\,m/s^2)$ $\boxed{a = 4.9\,m/s^2}$	

Block 1 | Block 2 | $a = 4.9\ m/s^2$

$t = ?$ | $-V = 40\ m/s$ | $V = 10\ m/s$ | $U_K = 0.5$

PART C: Calculate the time at which the 1 kg block comes completely to rest and the time at which the 4 kg block comes completely to rest on their respective rough surfaces.

1-kg Block		4-kg Block	
$\cancel{V_f}^{0} = V_i + at$ $t = \frac{-V_i}{a} = \frac{-(-40\ m/s)}{4.9\ m/s^2}$ $t = 8.2\ sec$	on Rough Surface Block slows to Stop Velocity (−) going to Left ∴ a is Positive	$\cancel{V_f}^{0} = V_i + at$ $t = \frac{-V_i}{a}$ $= \frac{-10\ m/s}{-4.9\ m/s}$ $t = 2.0\ s$	Velocity + (Right) ∴ a is −
$T_T = T_S + T_R$ $= 1 + 8.2$ $\boxed{T_T = 9.2\ se}$	Total time $T_T = T_{to\ Rough}^{(T_S)} + T_{to\ Stop\ on\ Rough}$	$T_T = T_{Smooth} + T_{Rough}$ $T_T = 4s + 2$ $\boxed{T_T = 4s}$	

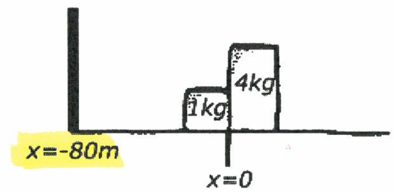
Center of mass Problem

wall

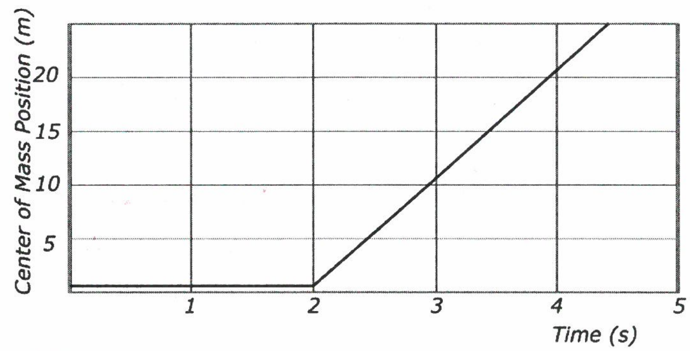
Argumentation

Skip

Now, the blocks are exploded apart the same way as before, but this time, friction is negligible everywhere on the track. Instead, there is a wall located at $x = -80$ m. Upon striking the wall, the 1 kg block sticks to the wall. The graph of the center of mass position of the two-block system is shown as a function of time.



Skip **PART D:** The graph at right shows the position of the center of mass of the two-block system as a function of time. The graph is zero for $0 < t < 2$ s and has a constant positive slope for $t > 2$ s. In a clear, coherent, paragraph-length response, explain why the graph has these features.



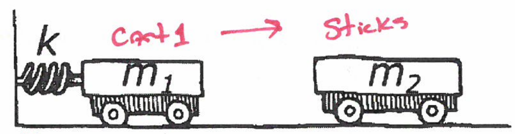
Checklist:

- ________ I answered the question directly.
- ________ I stated a law of physics that is always true.
- ________ I connected the law or laws of physics to the specific circumstances of the situation.
- ________ I used physics vocabulary (energy, mass, momentum, force, velocity, center of mass, time).

NAME ______________________ DATE Key

Scenario

Angela, Blake, Carlos, and Dominique are performing an experiment involving Cart 1 and Cart 2, which are both light and the friction in the bearings can be neglected. The students push Cart 1 against a spring (force constant k), compressing the spring a distance x from its equilibrium length. Cart 1 is released, collides with Cart 2, and sticks. The two carts continue with constant speed v_f after the collision. The students are tasked with making v_f as fast as possible but with the following constraints:



- *The spring compression distance x cannot be varied.*
- *Masses can be added or removed from either Cart 1 or 2 or both as long as the total mass of the system ($M = m_1 + m_2$) **remains constant**.*

$\boxed{m_1}\boxed{m_2}$ → V_f

want max V_f

Total M constant
But $M = \uparrow m_1 + \downarrow m_2$ or $\downarrow m_1 + \uparrow m_2$

Argumentation

PART A: Answer the following question. Explain your reasoning. You may cite equations but do not manipulate or combine equations as part of your explanation.

i. After Cart 1 is launched, how will the total mechanical energy of the system (spring and m_1), change if m_1 is large?

Claim: The Total ME (U_{spring} + KE) of System doesn't Change if M_1 is large

Evidence/Reasoning: There is no net work done (no friction) the Total ME is constant (until collision w/ m_2) $ME = \frac{1}{2}kx^2 + \cancel{KE}^{0}$, starts at rest

$v=$

ii. After Cart 1 is launched, how will the total momentum of the system change if m_1 is large?

Claim: If M_1 is larger, the Total momentum of M (combine mass) After it is launch will Be larger

Evidence/Reasoning:

The KE given to the Block is only Based on spring!!
$p = mv$, so momentum ↑ with increasing mass

PART B: Based on one or both of your answers to Part A, explain whether m_1 should be large or small to make the final speed v_f the fastest. goal) V_f max!

M_1 should be large to achieve max V_f.
↑m_1 doesn't affect ME total, but will ↑p a
the speed o M_1 before collision w/ m_2. This will
↑ V_f after the collision

Quantitative Analysis

PART C: Derive an expression for v_f, the combined cart speed, in terms of m_1, M, x, and k. Then explain how this expression supports your assertions in Part B.

(1) $ME_i = ME_f$ $\frac{1}{2}Kx^2 + \cancel{\frac{1}{2}mv_i^2}^{0} = \frac{1}{2}mV_2^2 + \cancel{U_g}^{0}$ Rest	Total ME before launch to after remains constant because there is no net external forces doing work on spring cart system
(2) $\cancel{\frac{1}{2}}Kx^2 = \cancel{\frac{1}{2}}mV_2^2$ $m_1V_2^2 = Kx^2$	Energy stored in spring is equal to KE of m_1 after launch
(3) $V_2 = \sqrt{\frac{Kx^2}{m_1}}$	The speed of M_1, after launch is equal to: as M_1↑ V_1↓
(4) $P_i = P_f$	During collision of 2 blocks, no net external forces, ∴ momentum conserved
(5) $m_1V_2 = (m_1+m_2)V_f$	The momentum of block 1 before collision is equal to momentum of the 2 blocks combined, after collision

(6) $m_1 v_2 = (m_1 + m_2) v_f$ $v_2 = \sqrt{\frac{Kx^2}{m_1}}$ $m_1 \sqrt{\frac{Kx^2}{m_1}} = (m_1 + m_2) V_f$	
(7) $m_1 \sqrt{\frac{Kx^2}{m_1}} = M V_f$ $V_f = \frac{m_1}{M} \sqrt{\frac{Kx^2}{m_1}}$	
(8)	

Line number 9 supports my claim by: if $m_1 \uparrow$ the final speed increases