

Workbook |

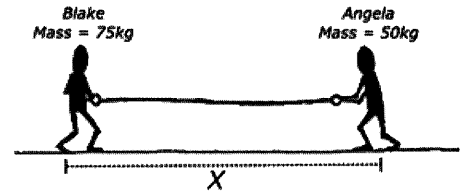
Unit 5 - Momentum

NAME _____

DATE _____

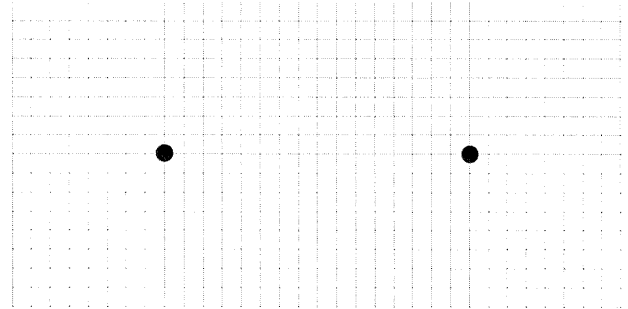
Scenario

Two students, Blake and Angela, stand on a smooth icy surface, a distance x apart and pull on opposite ends of a rope to pull themselves together. They each hold tightly onto the rope, which has negligible mass. Angela pulls on the rope with a constant force, so that she and Blake approach each other and meet. The system includes both students and the rope.



Using Representations

PART A: The dots to the right represent the two students. Draw free-body diagrams showing and labeling the forces (not components) exerted on each student. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.



Data Analysis

PART B: From the following four statements about the situation above, place a check mark next to the statement if it is completely true and provide justification. If it is partly true, correct the statement, and if it is false, cross out the statement and provide justification.

_____ A. Only Angela moves relative to the ice.

_____ B. The magnitude of Angela's acceleration is less than the magnitude of Blake's acceleration.

_____ C. Just before they meet, Blake's speed is less than Angela's speed.

_____ D. While the students are moving, their momentum vectors have equal magnitude and direction.

Argumentation

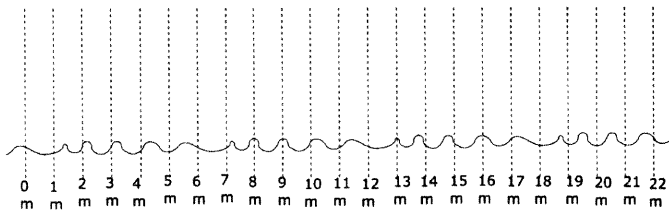
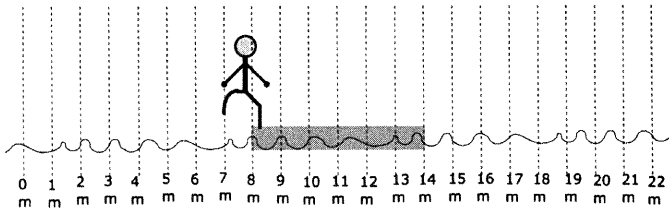
PART C: Where do the two students meet?

The students meet at the _____ of the system.

Justify your answer in a few short sentences. You do not need to do any calculations to determine where they meet.

5.A Center of Mass

PART D: Carlos ($m = 50 \text{ kg}$) stands on the far-left edge of a 100 kg stand-up paddle board. The board is 6 m long as shown in the diagram below and slides across the surface of the water with negligible friction. On the diagram, show the location of the student-board center of mass. Then draw what the system will look like after Carlos walks to the other end of the board. On this second diagram, mark the location of the student-board center of mass.

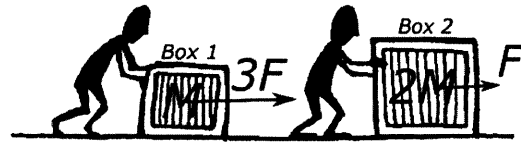


Briefly explain how you made your second drawing. How did you know where to set the board?
Give an explanation in terms of external forces and center-of-mass concepts.

NAME _____ DATE _____

Scenario

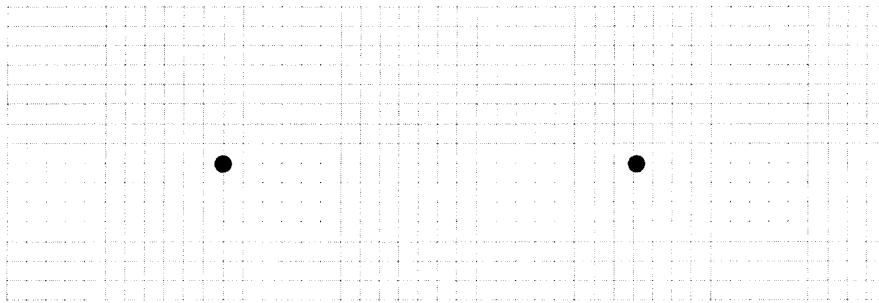
Two different boxes are pushed from rest on a frictionless surface for a time t , with differing forces.



Using Representations

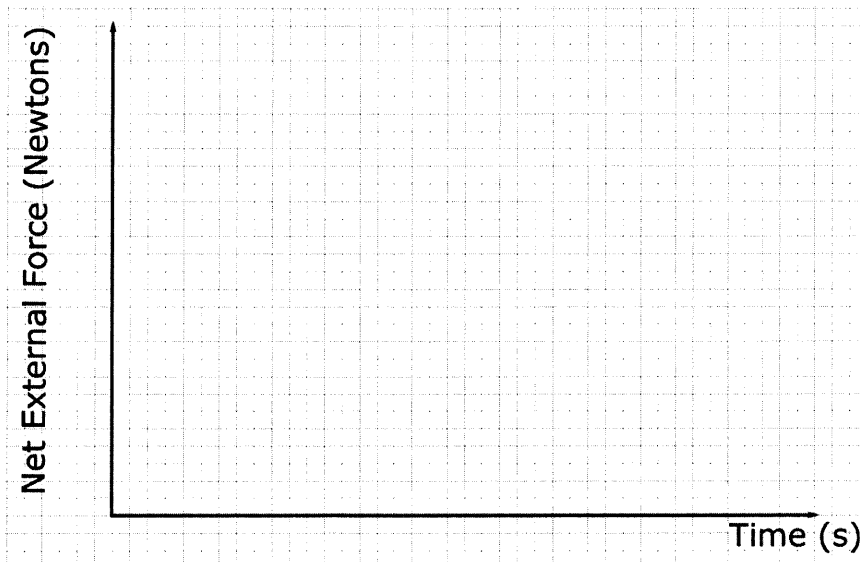
PART A: Identify the box system by circling the objects inside the system with a dotted circle.

PART B: The dots below represent the two boxes. Draw free-body diagrams showing and labeling the forces (not components) exerted on each box. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Identify the external forces.



Forces that are external to the system include: _____

PART C: Sketch a graph of the net external force vs. t for each box on the same axis by using two different colors and providing a key.



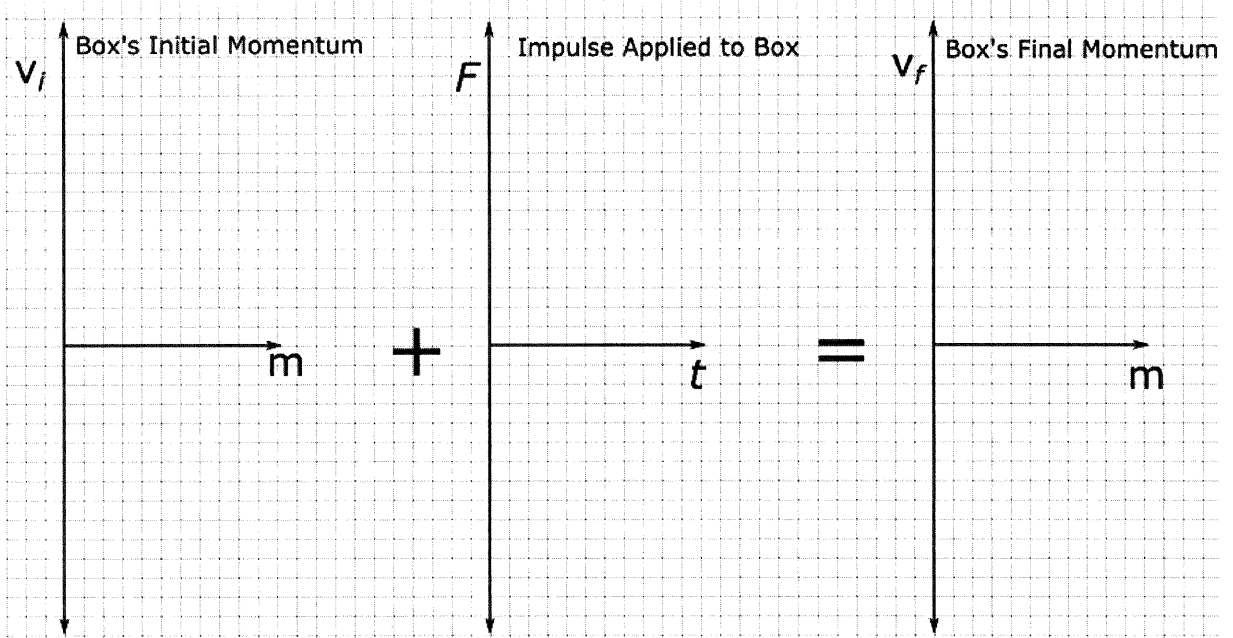
Quantitative Analysis

PART D: Using the equation for the area of a rectangle, $A = bh$, write two equations, one with words and one with units, for the area of the rectangle between the net external force vs. time line and the x -axis on the graph created in Part C.

The area under a net external force vs. time graph represents the _____ (Hint: Check units!)

5.B Impulse

PART E: Re-represent the data above by creating a momentum diagram for each box on the same set of axes by using two different colors and providing a key. (A momentum diagram represents the momentum of an object or system by showing the mass of the object as the width of bar on the chart, and the velocity of the object as the height of the bar.)



Argumentation

PART F: Is the work done on the first box while speeding up greater than, less than, or the same as the work done on the second box while speeding up?

_____ Greater than _____ Less than _____ Same

PART G: Is the impulse given to the first box while speeding up greater than, less than, or the same as the impulse given to the second box while speeding up? Justify your ranking.

_____ Greater than _____ Less than _____ Same

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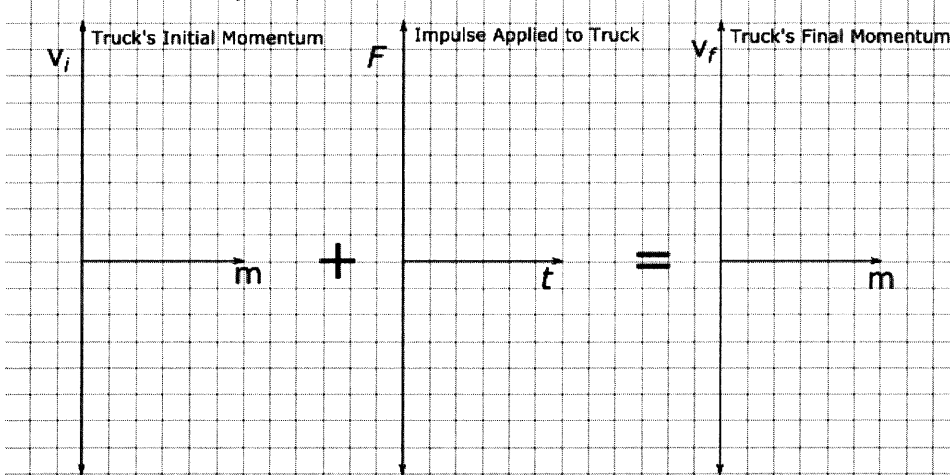
Scenario

A truck has a mass of M and initially moves with a speed of v_0 . Consider two cases: The truck makes a "gentle" stop, and the truck makes an "emergency" stop.

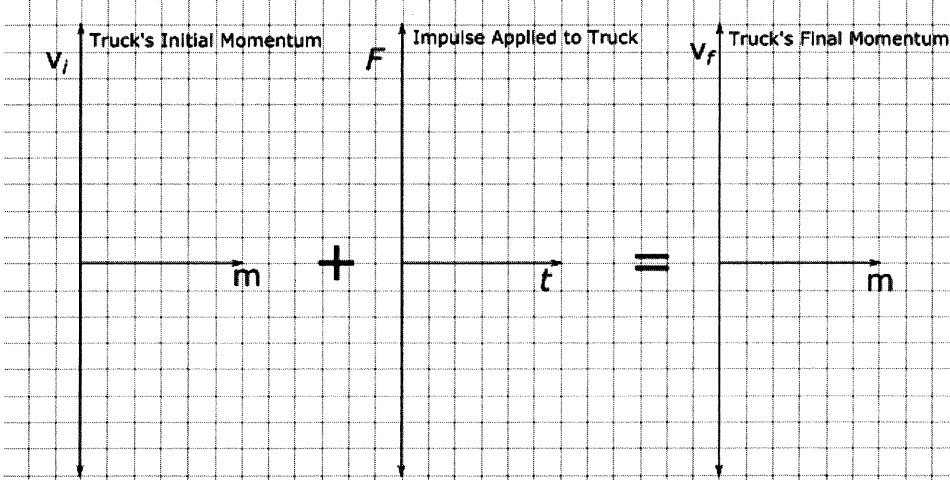
Using Representations

PART A: Show on the diagrams what a "gentle" stop and an "emergency" stop might look like.

Gentle Stop



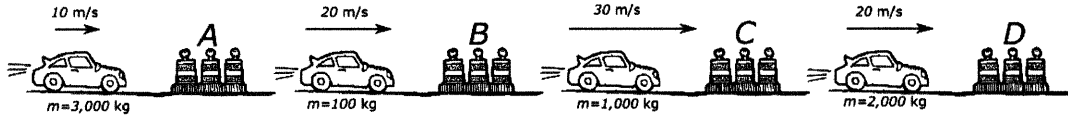
Emergency Stop



5.C Impulse

Argumentation

A car company tests its safety features by crashing cars into barriers in its testing facility. The cars are all the same size and shape but are moving at different speeds and have different masses. Assume that the barriers are all identical and exert the same constant force.



PART B: Rank the time it takes the cars to stop if the barriers apply the same constant force.

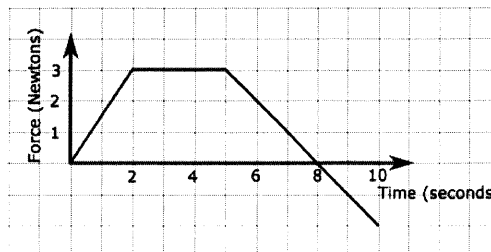
Longest time _____ Shortest time

Justify your ranking in a few short sentences.

NAME _____ DATE _____

Scenario

A 10 kg box, initially at rest, moves along a smooth horizontal surface. A horizontal force is applied to the box. The magnitude of the force changes as a function of time as shown. Take the positive direction to be to the right.



Data Analysis

PART A: Rank the magnitude of the impulse applied to the box by the force during each 2-second interval indicated below:

- A. 0–2 seconds B. 2–4 seconds C. 4–6 seconds D. 6–8 seconds E. 8–10 seconds

Greatest impulse _____ Smallest impulse _____

PART B: Write a few sentences justifying your reasoning. Use words like speed, velocity, acceleration, time, force, momentum, and impulse.

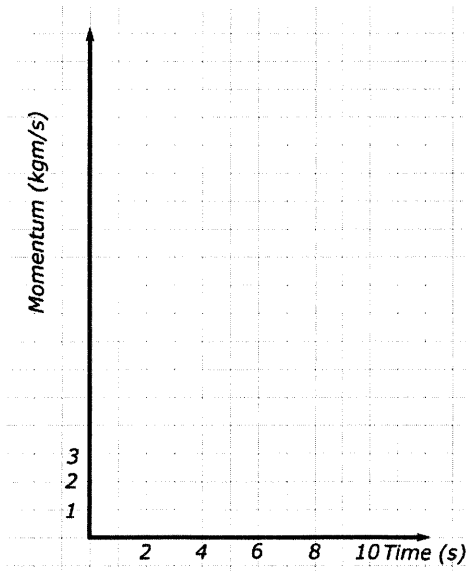
Using Representations

PART C: Re-represent the data given in the force vs. time graph in Part A as a momentum vs. time graph for the same 10 kg box.

Step 1: Identify the equation that relates force and momentum.

Step 2: How can momentum be found from a force vs. time graph?

Step 3: Plot the momentum as a function of time. (Make a table if you need to.)



5.D Change in Momentum

Quantitative Analysis

Use the graph above to calculate the velocity of the box after 10 seconds.

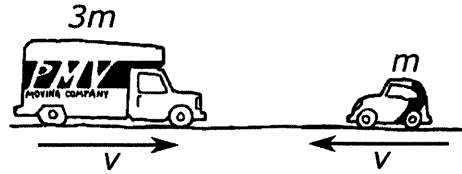
For each line in the calculation, explain what was done mathematically. The first line is done for you.

$\Delta \vec{p} = m\Delta \vec{v}$	The change in momentum is equal to the mass times the change in velocity.

NAME _____ DATE _____

Scenario

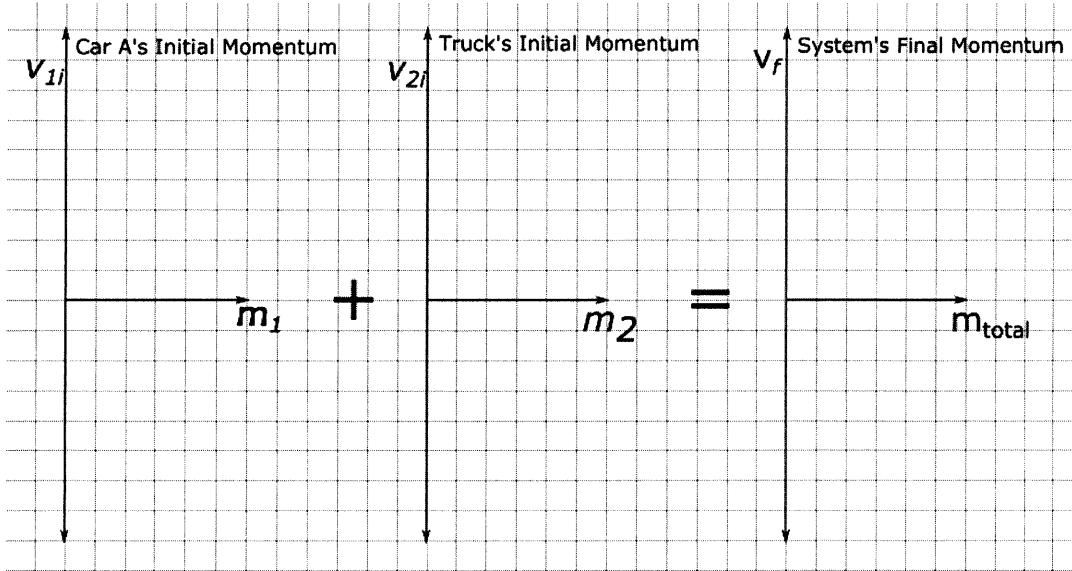
A toy car of mass m and a toy truck with a mass $3m$ travel in opposite directions at identical speeds. The truck moves to the right and the car moves to the left. The two toys collide and stick together.



Using Representations

PART A: Identify the system by drawing a dotted circle around the truck and the car.

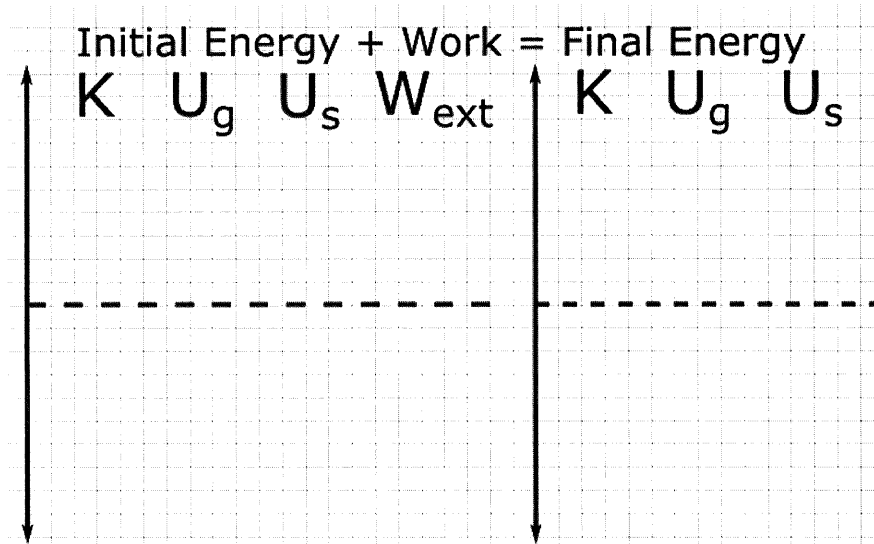
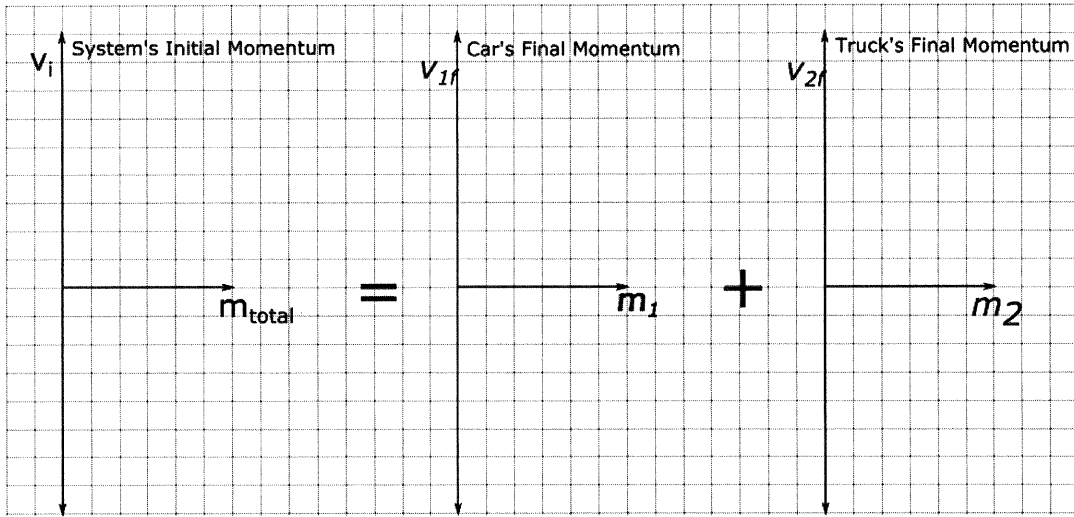
PART B: Diagram the situation.



In which direction will they be traveling after the they collide? Explain and justify your answer.

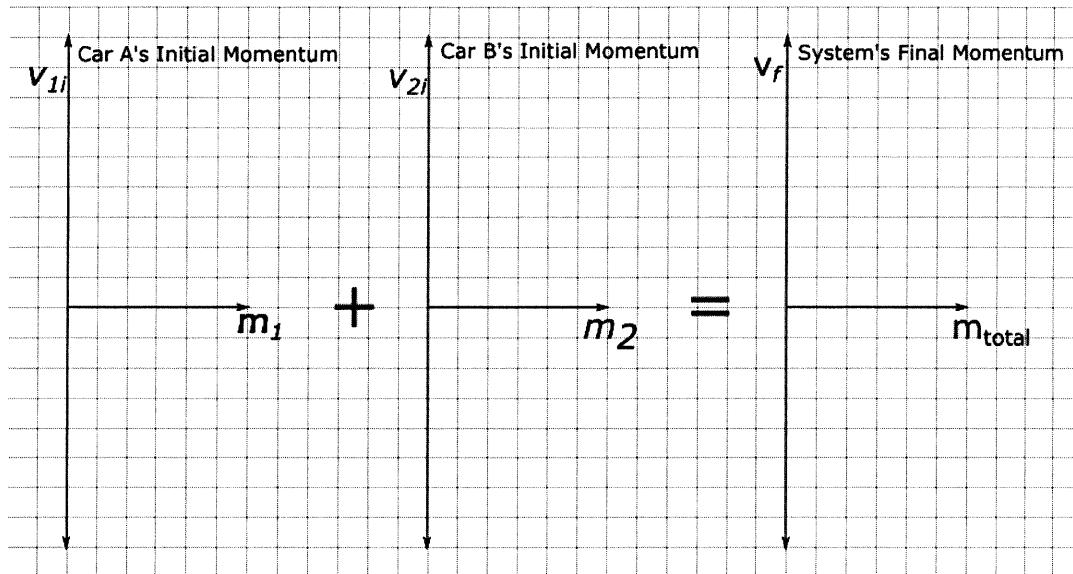
5.E Conservation of Momentum (Inelastic Collisions)

PART C: The toy car and truck are now pressed together with an ideal spring compressed between them. They are then released from rest. Diagram the momentum before and after the explosion as well as the energy before and after the explosion.



5.E Conservation of Momentum (Inelastic Collisions)

PART D: If there were instead two identical toy cars traveling in opposite directions at identical speeds, how would the momentum diagram change, and what direction would they be traveling after the collision? Explain and justify your answer.

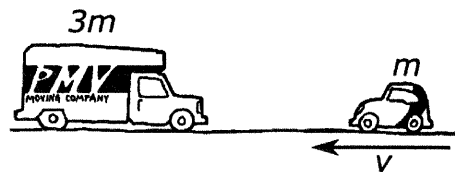


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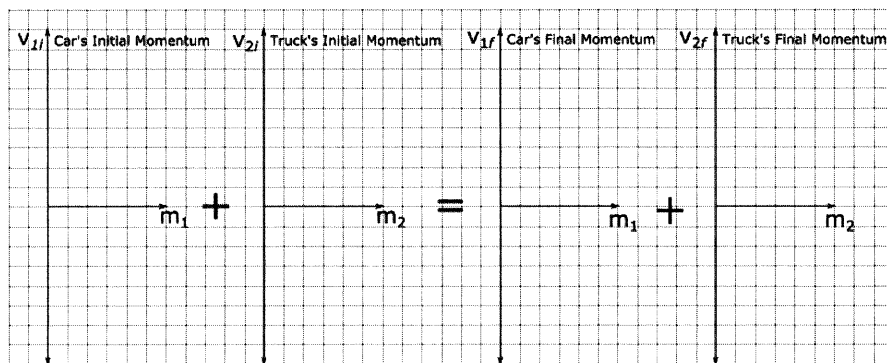
Scenario

A toy car is pushed with a speed v toward a toy truck initially at rest. The car bounces back off the truck so that the car's final speed is $\frac{v}{2}$ in the opposite direction. Consider the system to be the car and the truck.



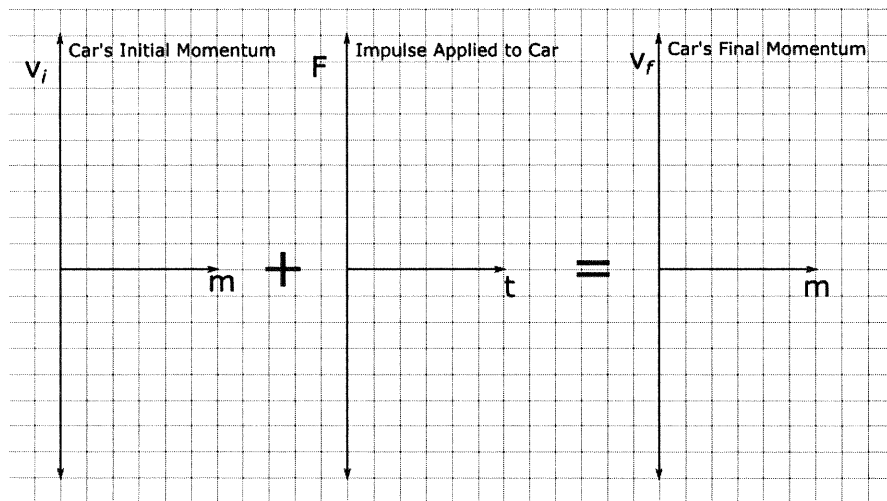
Using Representations

PART A: Sketch a momentum diagram for the car and truck before and after the collision.



How does the situation change if we consider the system to only contain the car?

PART B: Sketch a momentum diagram for the collision for the car-only system.



Data Analysis

PART C: In the laboratory, a ball is dropped onto a force-sensing platform several times, each time hitting a different surface (foam, feathers, clay, etc.). The momentum of the ball changes by the same amount in each trial; in each trial, the average scale reading is F , and the time of collision t are measured. What quantities would need to be graphed to exhibit a straight-line relationship?

Justify your answer in a few sentences.

Checklist:

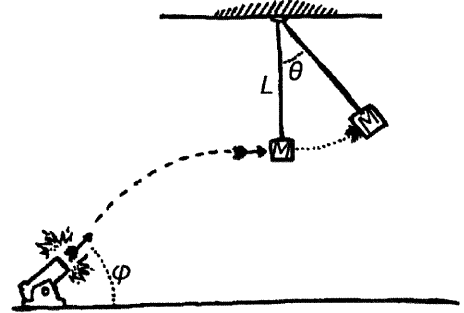
- I answered the question directly.
- I stated a law of physics that is always true.
- I connected the law or laws of physics to the specific circumstances of the situation.
- I used physics vocabulary (momentum, mass, energy, force, velocity, speed, time).

NAME _____

DATE _____

Scenario

A small dart of mass m is launched at an angle ϕ above the horizontal with initial speed v_0 . At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass M that is suspended at the end of a string of negligible mass. The center of the block is a distance L below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical as shown at the right. Air resistance is negligible.



Using Representations

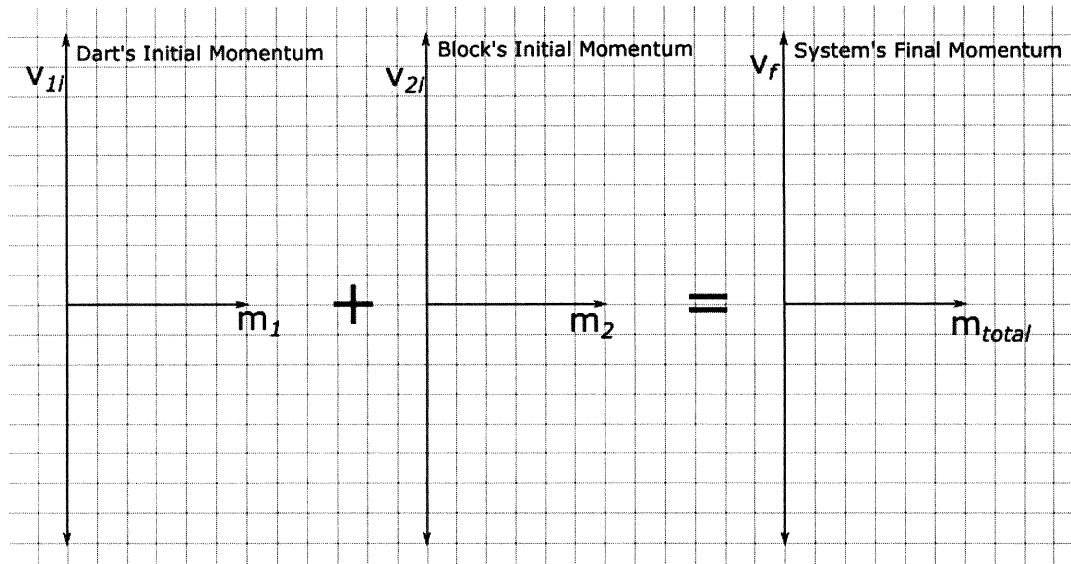
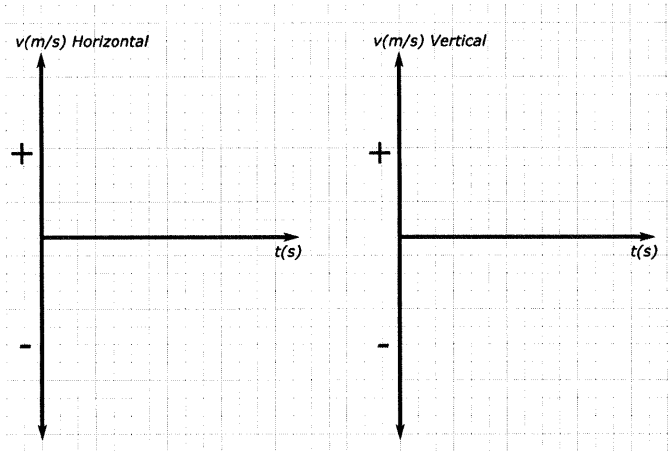
PART A:

- i. Sketch a horizontal and a vertical velocity vs. time graph from the time the dart is launched to the time it hits the block.
- ii. Determine the speed of the dart just before it strikes the block. Your answer should only include v_0 , ϕ , θ , L , and physical constants.

$v =$ _____

PART B:

- i. Identify the system by circling both the dart and the box with a dotted circle.
- ii. Sketch a momentum diagram for the system from just before to just after the dart collides with the block.



Argumentation

PART C: In a second experiment, a dart with a greater mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block. Would the angle theta (θ) that the dart and block swings to increase, decrease, or stay the same? In a clear, coherent paragraph-length response, that *may* also contain figures and/or equations, justify your choice.

_____ Increase _____ Decrease _____ Stay the same

Checklist:

_____ I answered the question directly.

_____ I stated a law of physics that is always true.

_____ I connected the law or laws of physics to the specific circumstances of the situation.

_____ I compared the situation (stated what was the same in all cases).

_____ I contrasted the situations (stated what was different in all cases).

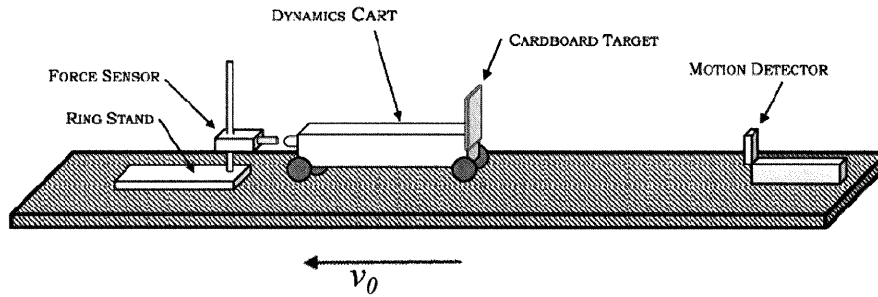
_____ I used physics vocabulary (momentum, mass, energy, force, velocity, speed, potential, kinetic, time).

NAME _____

DATE _____

Scenario

Carlos and Dominique have been challenged to design an experiment to determine the impulse given to a cart as it collides with a barrier using two different mathematical or graphical ways. Dominique sketches the following diagram of the laboratory setup and writes the procedure below.



Materials: computer, force sensor, motion sensor, cart, cardboard target, mass balance

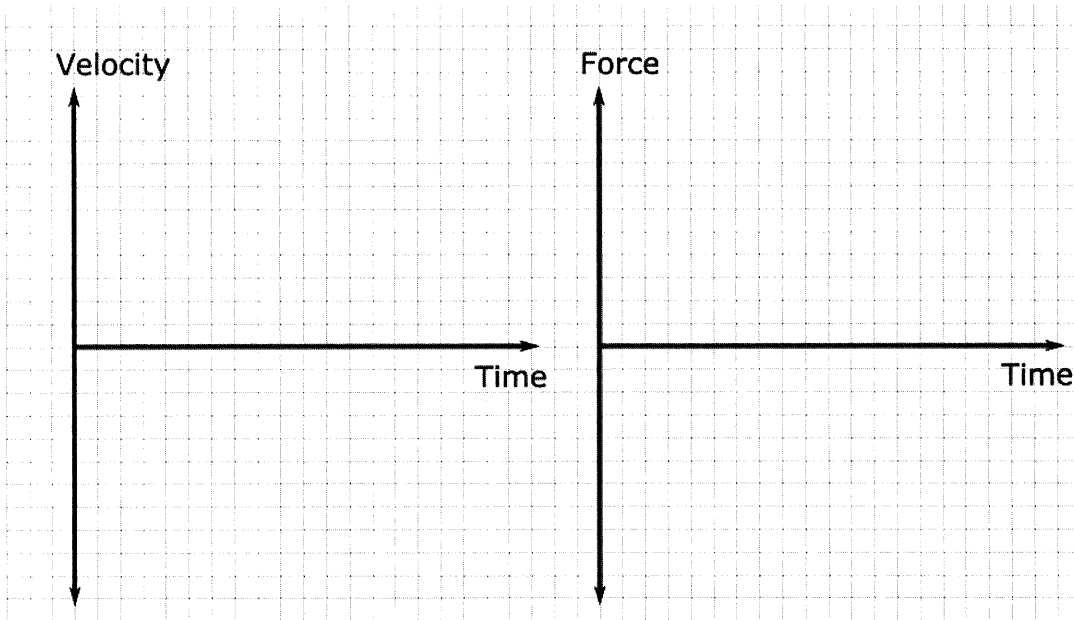
Experimental Design

PART A: Edit Dominique's procedure for length and clarity. Cross out any unnecessary statements, change the order of statements, correct statements for errors, or write new sentences if necessary.

1. Gather all materials.
2. Record the mass of the cart.
3. Plug in both the motion detector and the force sensor.
4. Check that each device is working.
5. Secure the motion detector to the ring stand.
6. Attach the cardboard target to the cart so that it can be "seen" by the motion sensor.
7. Align the motion sensor with the target cardboard.
8. Create a data table in your notebook.
9. Set the motion sensor to record in cart mode.
10. Begin recording force and motion data with the computer.
11. Give the cart a push toward the force sensor and away from the motion detector.
12. After the cart collides with the force sensor and has bounced back, stop recording force and motion data.
13. Determine the impulse.
14. Repeat steps 9–12 with different initial pushes to reduce error.
15. Clean up the lab station and put away all materials.

Using Representations

PART B: Sketch a graph of what the velocity as a function of time and the force as a function of time should look like for the time just before the collision to just after the collision.



Data Analysis

PART C: Explain how these representations can be used to determine the impulse given to the cart during the collision. Explain how you could determine the impulse given to the cart from each graph.

NAME _____

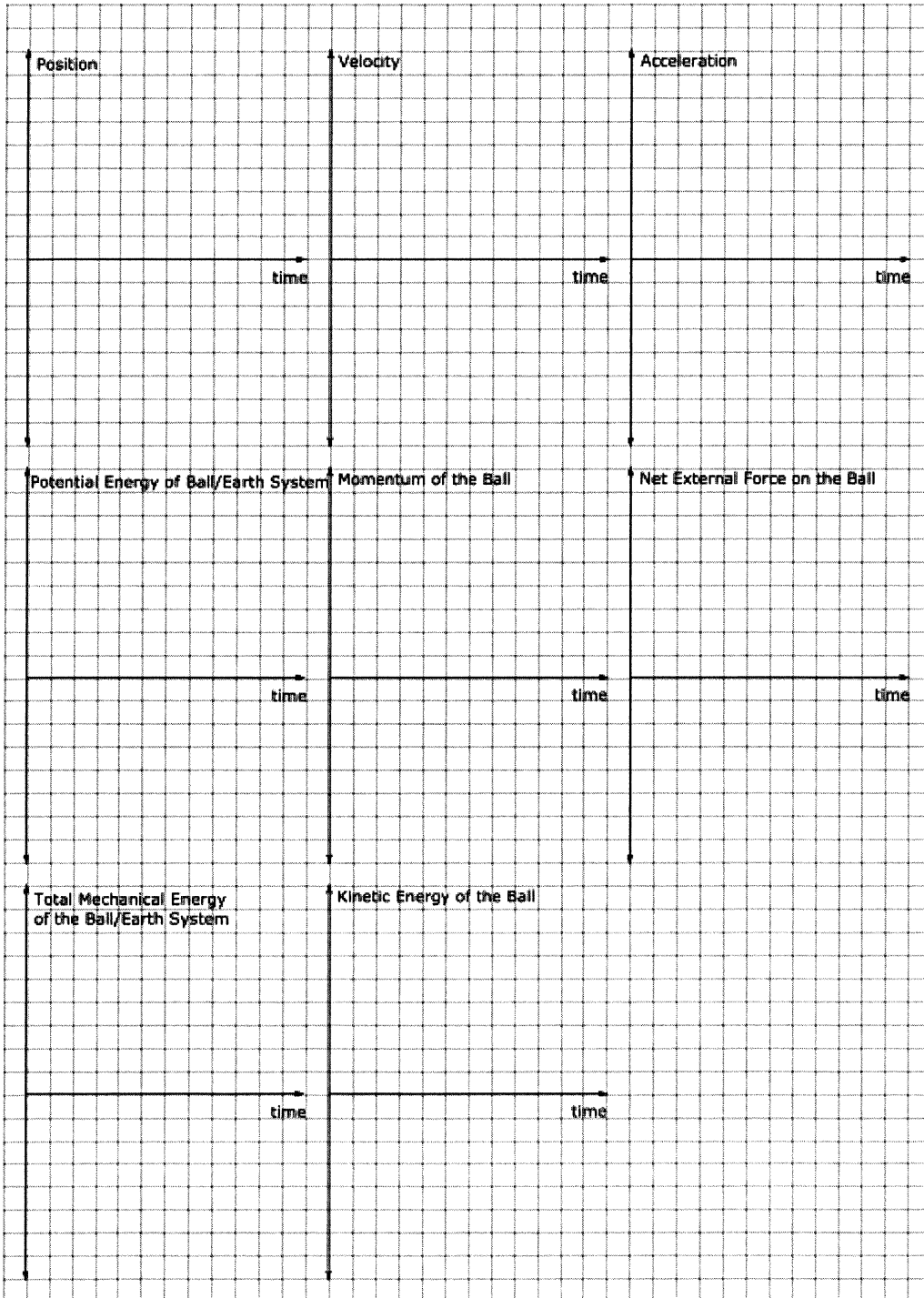
DATE _____

Scenario

A ball is thrown straight up into the air with an initial speed v_0 . After a few seconds, it returns to the height from which it was thrown. Air resistance is negligible.

Using Representations

PART A: Sketch the following graphs as functions of time for the time the ball is in the air.



5.I Momentum Representations

Data Analysis

PART B: The following sentences discuss the relationships between these graphs and the physical ideas they represent. Fill in the blanks.

1. The slope of the position vs. time graph is equal to _____.
2. The slope of the velocity vs. time graph is equal to _____.
3. The area under the acceleration vs. time curve is equal to the _____.
4. The area under the velocity vs. time curve is equal to the _____,
5. The graph of momentum vs. time is the same shape as the _____ vs. time graph because momentum is equal to _____.
6. The net force graph vs. time is the same shape as the _____ vs. time graph because the net force is equal to _____.
7. The slope of the momentum vs. time graph is equal to _____.
8. The area under the curve of the net external force vs. time graph is equal to _____ or _____.
9. The potential energy vs. time graph is the same shape as the _____ vs. time graph because the potential energy is equal to _____.
10. The kinetic energy vs. time graph is related to the _____ vs. time graph because the kinetic energy is equal to _____.
11. The total mechanical energy graph is _____ because it represents the sum of the _____ vs. time and the _____ vs. time graphs.
Also, there are no _____ forces on the system, so there is no work done. Therefore, the total mechanical energy is _____.

NAME _____ DATE _____

Scenario

Angela and Carlos have identical masses M and stand on identical light carts with bearings with negligible friction. Carlos holds a heavy ball of mass m . At time $t = 0$, Carlos throws the ball so that it has a horizontal component of velocity v while a projectile. At time $t = T$, Angela catches the ball at the same height from which it was thrown.



Argumentation

PART A: Which student moves with faster speed for time $t > T$? Explain your reasoning *qualitatively* without manipulating equations.

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Quantitative Analysis

PART B: Derive expressions for v_{fA} and v_{fC} , Angela and Carlos's final speeds for time $t > T$ in terms of M , m , and v .

	System is Carlos and ball. There are no net horizontal external forces, so the momentum of the system is conserved.
	Initially, Carlos and the ball are at rest, so their initial momentum is zero, so their final momentum must also be zero.
	Equation for v_{fC} in terms of M , m , and v .
	For the second part of the problem, the system is the ball and Angela. There are no net horizontal external forces, so the momentum of the system is conserved.
	Initially, Angela is at rest and the ball has momentum mv , which means that the final momentum must also equal mv . After the collision, the ball and Angela have a new velocity v_{fA} .
	Equation for v_{fA} in terms of M , m , and v .

Data Analysis

PART C: Explain how your expressions in Part B support your reasoning in Part A.

PART D: Let E_1 be the mechanical energy of the Angela-Carlos-ball system for $t < 0$, let E_2 be the mechanical energy of this system for $0 < t < T$, and let E_3 be the mechanical energy of this system for $t > T$. Rank these energies from highest to lowest and explain your reasoning.

Highest energy _____ Lowest energy

Checklist

- _____ I answered the question directly.
- _____ I stated a law of physics that is always true.
- _____ I connected the law or laws of physics to the specific circumstances of the situation.
- _____ I used physics vocabulary (energy, mass, momentum, conservation, velocity, time).

NAME _____

DATE _____

Scenario

Blake is given two carts of unequal masses, m and M , and a long track that ensures both carts travel in a straight line with no noticeable friction. He also has access to other commonly available equipment. The carts do NOT have any way to connect together when they come into contact. He asks three questions:

1. When the two carts collide, is the total momentum of the system conserved?
2. When the two carts collide, is the mechanical energy of the two-cart system conserved?
3. When the two carts collide, is the force m exerts on M equal to the force that M exerts on m ?

Experimental Design

PART A: Outline a procedure that Blake could follow to make measurements that could be used to answer all three questions above. Give each measurement a meaningful algebraic symbol and state with what equipment each measurement is made. Draw a labeled diagram showing each piece of equipment being used.

What Needs to Be Measured and Algebraic Symbols	Procedure:
Labeled Diagram of the Setup	

Data Analysis

PART B:

i. Explain how the measurements made in Part A can be used to answer question 1.

ii. Explain how the measurements made in Part B can be used to answer question 2.

iii. Explain how the measurements made in Part A can be used to answer question 3.

NAME _____

DATE _____

Scenario

A dart launcher fires a dart so fast that the speed cannot be measured by any direct method. Angela wishes to determine the speed with which the dart is fired. After being fired, the dart will embed itself into a wooden block. She already knows the mass m of the dart.

Experimental Design

PART A: Angela has access to the equipment listed below. Mark the space next to each piece of equipment that she could use in an experiment to determine the dart's speed.

_____ Meterstick _____ Stopwatch _____ String _____ Electronic balance

_____ Cart with frictionless bearings and light wheels (that the wood block can be attached to)

_____ Track (that allows the cart to travel in a straight line with no noticeable friction)

_____ Hooks (that can be connected to the ceiling of the classroom or the block)

_____ Camera (that can take video that can be replayed frame by frame with time codes indicated on each frame)

PART B: Outline a procedure that Angela could follow to use the equipment that you marked above to make measurements that could be used to calculate the firing speed of the dart. Give each measurement a meaningful algebraic symbol. Draw a labeled diagram of the experimental setup.

What Needs to Be Measured and Algebraic Symbols	Procedure:
Labeled Diagram of the Setup	

5.L Inelastic Collisions

Data Analysis

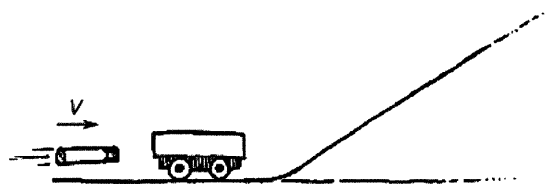
PART C: Explain how the measurements can be used to calculate the speed with which the dart is fired from the gun. As part of your explanation, show symbolic equations that measurements must be plugged into and solved to arrive at your conclusion.

NAME _____

DATE _____

Scenario

A wooden cart of mass M is set on a horizontal section of track. The cart experiences negligible friction as it rolls. A dart of mass $m < M$ is fired with initial speed v toward the cart, which is initially at rest.



Consider the following cases:

Case 1: The dart embeds itself in the cart.

Case 2: The dart bounces backward off the cart.

Case 3: The dart passes through the cart.

Using Representations

PART A: For each case, create a pictorial representation of conservation of momentum during the collision.

<p>Dart's Initial Momentum</p> <p>v_{1i}</p> <p>m_1</p>	+	<p>Cart's Initial Momentum</p> <p>v_{2i}</p> <p>m_2</p>	=	<p>System's Final Momentum</p> <p>v_f</p> <p>m_{total}</p>		
<p>Dart's Initial Momentum</p> <p>v_{1i}</p> <p>m_1</p>	+	<p>Cart's Initial Momentum</p> <p>v_{2i}</p> <p>m_2</p>	=	<p>Dart's Final Momentum</p> <p>v_{1f}</p> <p>m_1</p>	+	<p>Cart's Final Momentum</p> <p>v_{2f}</p> <p>m_2</p>
<p>Dart's Initial Momentum</p> <p>v_{1i}</p> <p>m_1</p>	+	<p>Cart's Initial Momentum</p> <p>v_{2i}</p> <p>m_2</p>	=	<p>Dart's Final Momentum</p> <p>v_{1f}</p> <p>m_1</p>	+	<p>Cart's Final Momentum</p> <p>v_{2f}</p> <p>m_2</p>

Quantitative Analysis

PART B: Create a mathematical representation showing conservation of momentum for each of the three cases. Use each representation to derive the velocity v_{2f} of the cart after the collision.

<i>Case 1</i>	<i>Case 2</i>	<i>Case 3</i>

Argumentation

PART C: Rank the cases in terms of the distance up the incline that the cart travels after its interaction with the dart and explain your ranking in terms of conservation of momentum and conservation of energy.

Claim: Farthest up the incline _____, _____, _____ Least far up the incline

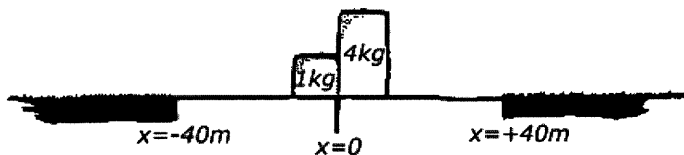
Evidence/Reasoning: (Use evidence and reasoning from Parts A and B to support your claim.)

NAME _____

DATE _____

Scenario

Two blocks of mass 1 kg and 4 kg are placed in contact at the center point of an 80 m long track that exerts negligible friction on the blocks. At both ends of the track, there are strips of rough surface of equal coefficient of kinetic friction 0.5. A small explosive charge between the blocks propels them apart at time $t = 0$ so that the 1 kg block moves with a speed of 40 m/s along its section of the frictionless track.



(This figure is not to scale; the size of the two blocks is significantly smaller than the distance to the location where the frictionless track is connected to a rough surface.)

Quantitative Analysis

PART A: Calculate the time at which the 1 kg block reaches its rough surface and the time at which the 4 kg block reaches its rough surface. Verbally explain your calculations.

1-kg Block

4-kg Block

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5.N Center of Mass Motion

1-kg Block

4-kg Block

PART B: Both blocks have the same magnitude of acceleration while sliding to rest on their respective rough surfaces. Calculate this acceleration and verbally explain your method.

5.N Center of Mass Motion

PART C: Calculate the time at which the 1 kg block comes completely to rest and the time at which the 4 kg block comes completely to rest on their respective rough surfaces.

1-kg Block		4-kg Block	

NAME _____

DATE _____

Scenario

Angela, Blake, Carlos, and Dominique are performing an experiment involving Cart 1 and Cart 2, which are both light and the friction in the bearings can be neglected. The students push Cart 1 against a spring (force constant k), compressing the spring a distance x from its equilibrium length. Cart 1 is released, collides with Cart 2, and sticks. The two carts continue with constant speed v_f after the collision. The students are tasked with making v_f as fast as possible but with the following constraints:



- The spring compression distance x cannot be varied.
- Masses can be added or removed from either Cart 1 or 2 or both as long as the total mass of the system ($M = m_1 + m_2$) remains constant.

Argumentation

PART A: Answer the following question. Explain your reasoning. You may cite equations but do not manipulate or combine equations as part of your explanation.

- i. After Cart 1 is launched, how will the total mechanical energy of the system (spring and m_1), change if m_1 is large?

Claim: _____

Evidence/Reasoning: _____

- ii. After Cart 1 is launched, how will the total momentum of the system change if m_1 is large?

Claim: _____

Evidence/Reasoning: _____

5.O Conservation of Energy and Momentum

PART B: Based on one or both of your answers to Part A, explain whether m_1 should be large or small to make the final speed v_f the fastest.

Quantitative Analysis

PART C: Derive an expression for v_f , the combined cart speed, in terms of m_1 , M , x , and k . Then explain how this expression supports your assertions in Part B.

(1)	
(2)	
(3)	
(4)	
(5)	

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(6)	
(7)	
(8)	

Line number _____ supports my claim by: _____
