AP Physics 1

Unit 6: Rotation

6.1– Rotational Kinematics

Focus Question: How is rotational kinematics different from linear kinematics?

Review: *Kinematics* - **study of motion without regards to forces causing it.**

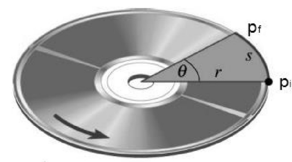
Torque when dealing with rotation

Revolution – Circular motion of an object about an axis that does not lie inside the object.

ex: a roller coaster going around a loop, Earth orbiting the sun.

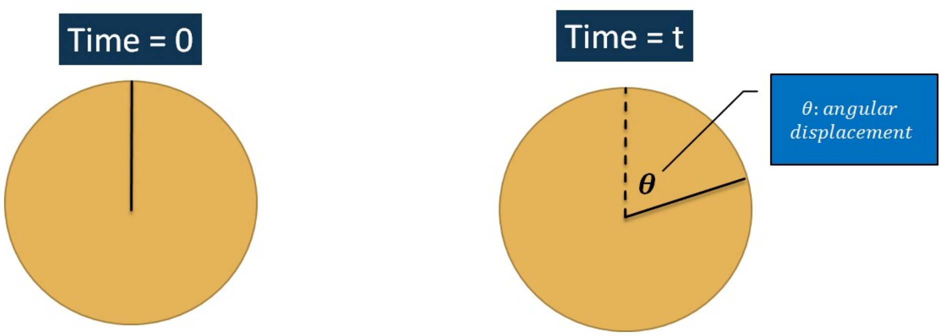
Rotation - Spinning of an object about an axis that lies within the object.

ex: a wheel's spin, the rotation of Earth about its axis, CD disc spinning



Describe kinematic quantities in rotational motion and relate them to linear motion.

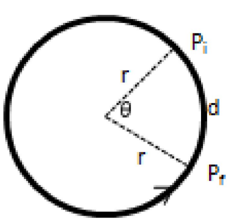
Angular Displacement - Vector quantity representing how much a rotating object spins (the angle)



Units of angular displacement: ***radians (rad)***

$$\pi \, rad = 180°$$

Relationship between linear distance and angular displacement:



Linear displacement vs. angular displacement

- ***Linear displacement is the length of the arc of the path of an object a radius r from the center of rotation.***
- ***From math:*** $s = r\theta$

Example A: A hipster spins a vinyl copy of an album you've probably never heard of on their vintage record player. The record spins 10 times.

a) Find the angular displacement of the record in that time.

There are 2π radians for every revolution.

$$\theta = 2\pi \frac{rad}{rev} * 10\, rev = \boldsymbol{20\pi\, rad}$$

b) While the record was spinning, a fly stands on the record 10 cm from the center of the disc. Find the length of the fly's path while the record is spinning.

$$s = r\theta = (.10\, m)(20\pi) = \boldsymbol{6.3\, m}$$

Everything on a spinning axis has the same angular displacement. However, linear distance traveled is the based on the distance from the center of rotation. Objects with a greater radius will travel a greater distance (arc length) the further they are from the center.

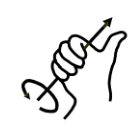
Angular Velocity - Rate of angular displacement (how fast an object is spinning)

Angular velocity : $\omega = \frac{angular\ displacement}{time} = \boldsymbol{\frac{\Delta\theta}{t}}$

Units of angular velocity: $\boldsymbol{rad/s}$

*Radians can be omitted when writing units so 1/s is also appropriate for velocity.

Vector direction (Right-hand rule): Curl the fingers of your right hand in the direction of motion, the direction of the vector is the direction your thumb points.



Relationship between linear speed and angular velocity- divide both sides of the relationship between linear displacement and angular displacement by time:

$$\boldsymbol{v = r\omega}$$

Angular Acceleration - Rate of change of angular velocity. For an object rotating in the positive direction, positive angular acceleration means it is spinning faster and faster and negative acceleration means the object's spin is slowing down.

$$\boldsymbol{angular\ acceleration = \alpha = \frac{\Delta\omega}{t}}$$

$$\boldsymbol{\alpha = \frac{\omega_f - \omega_f}{t}}$$

Units of angular acceleration: $\boldsymbol{rad/s^2}$ **or** $\boldsymbol{1/s^2}$

Relationship between linear speed and angular velocity-

$$\boldsymbol{a = r\alpha}$$

Kinematics Equations

Linear Equations:

$$\Delta x = v_i t + \frac{1}{2}at^2$$

$$\Delta x = v_f t - \frac{1}{2}at^2$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f = v_i + at$$

$$\Delta x = \left(\frac{v_i+v_f}{2}\right)t$$

Rotational Equations:

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \omega_f t - \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$\omega_f = \omega_i + \alpha t$$

$$\Delta\theta = \left(\frac{\omega_i+\omega_f}{2}\right)t$$

Apply the kinematics equations to rotational motion.

Example B: A wheel making 800 rev/min slows down to 500 rev/min while making 60 revolutions.

a) What is the angular acceleration?

b) How long does it take the wheel to slow down?

c) Find the linear speed of an object .35 m from the center of the wheel when the wheel is spinning at 500 rev/min.

$$\omega_i = 800 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{80\pi}{3} \frac{\text{rad}}{\text{s}}$$

$$\omega_f = 500 \frac{\text{rev}}{\text{min}} \times \frac{2\pi}{60} = \frac{50\pi}{3} \frac{\text{rad}}{\text{s}}$$

$$\theta = 60 \text{ rev} \times \frac{2\pi \text{ rad}}{\text{rev}} = 120\pi \text{ rad}$$

a) $$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta} = \frac{\left(\frac{50\pi}{3}\frac{1}{s}\right)^2 - \left(\frac{80\pi}{3}\frac{1}{s}\right)^2}{2(120\pi \text{ rad})} = -5.7 \text{ rad/s}^2$$

b) $$\omega_f = \omega_i + \alpha t \Rightarrow t = \frac{\omega_f - \omega_i}{\alpha} = \frac{\frac{50\pi}{3}\frac{1}{s} - \frac{80\pi}{3}\frac{1}{s}}{-5.7 \frac{1}{s^2}} = \underline{5.5 \text{ s}}$$

c) $$v = r\omega = (.35 \text{ m})\left(\frac{50\pi}{3} \text{ rad/s}\right) = 18 \text{ m/s}$$

Example C: The angular acceleration of a wheel is 4.0 rad/s^2. How many revolutions does the wheel make in the first 6.0 seconds if it is initially at rest?

$\alpha = 4.0 \ \frac{1}{s^2}$

$t = 6 \ s$

$\omega_i = 0$

$\Delta\theta = ?$

$$\Delta\theta = \cancel{\omega_i t}^{\,0} + \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(4 \ \frac{1}{s^2}\right)(6s)^2 = 72 \text{ rads}$$

$$\text{\# of rotations} = \frac{72 \text{ rads}}{2\pi \ \text{rad/rev}} = 11.4 \text{ revolutions}$$

$\Rightarrow$ 11 complete revolutions

Example D: A wheel is rotating at 4.0 rad/s. Its rotation accelerates at a rate of .25 rad/s^2.

a) What is the angular displacement in radians during the first 3.0 seconds?

b) What is the angular velocity at the end of 3.0 seconds?

c) What is the linear distance traveled by an object .40 m from the center of the wheel?

$\omega_i = 4 \ \frac{1}{s}$

$\alpha = .25 \ \frac{1}{s^2}$

$t = 3 \ s$

a) $\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \left(4 \ \frac{1}{s}\right)(3s) + \frac{1}{2}\left(.25 \ \frac{1}{s^2}\right)(3s)^2 = \underline{13.1 \text{ rads}}$

b) $\omega_f = \omega_i + \alpha t = \left(4 \ \frac{1}{s}\right) + \left(.25 \ \frac{1}{s^2}\right)(3s) = \underline{4.75 \ \frac{1}{s}}$

c) $s = r\theta = (.4 \ m)(13.1 \text{ rads}) = \underline{5.2 \ m}$

Rate your understanding: Rotational Kinematics

0	1	2	3	4
Wow, this is awkward. What was kinematics again?	I understand rotation and solve some of the problems.	I can set up and solve rotational kinematics problems with only a few errors.	I can set up and solve rotational kinematics problems with no errors.	I can explain and teach the concepts behind rotation.

6.2– Equilibrium of Rigid Bodies (Torque)

Focus Question: When is in object in static equilibrium?

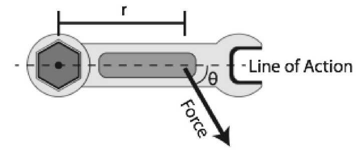
- **Rigid Body - An object that does not deform.**

For a rigid body to be in static equilibrium, the object must be in both **translational** equilibrium and **rotational** equilibrium.

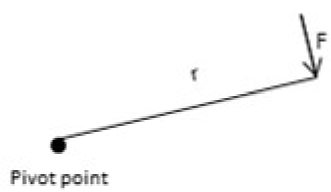
- Translational Equilibrium – **no linear acceleration** $(\sum \boldsymbol{F} = \mathbf{0})$
- Rotational Equilibrium - **no rotational acceleration** $(\sum \boldsymbol{\tau} = \mathbf{0})$

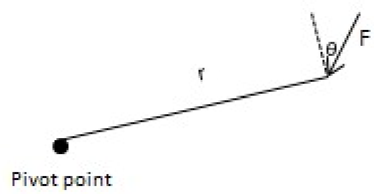
Calculate torque acting on an object.

- Torque – Measure of the force that makes an object rotate.
  Torque depends on both magnitude of the force and the distance from the axis of rotation.



- Magnitude of torque – Torque is based off the force applied and its distance to the axis of rotation.





If force is perpendicular to lever arm:

$$\boldsymbol{\tau = Fr}$$

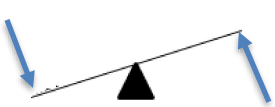
If force is not perpendicular

$$\boldsymbol{\tau = Fr \sin\theta}$$

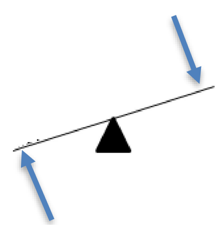
In path, torque is defined as the vector (cross) product of F and r: $\vec{\tau} = \vec{F}x\vec{r}$

*only the component of forces perpendicular to the axis of rotation causes rotation.

- Direction – Torque is a vector quantity, which means it has direction and magnitude. The directions of torque are clockwise or counter wise.

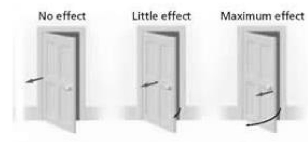


Counter-clockwise (positive) torque



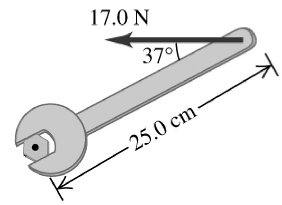
Clockwise (negative) torque

- Torque/distance relation: A force applied further away from the center of rotation creates a greater torque.
  ex: It's easier to open an open by pushing on the side opposite the hinges rather than near the hinges of the door.



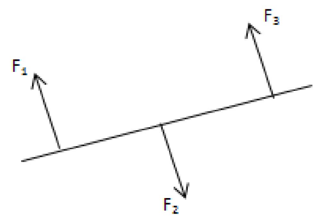
- Units of torque: Nm
  *torque has no relationship to work.

Example A: Bob the Builder uses a wrench to tighten a bolt as shown. He applies a 17.0 N force 25 cm from the bolt an angle of 37° with the axis of rotation. Calculate the magnitude and direction of the torque on the bolt.



$\tau = Fr\sin\theta = (17\ N)(.25m)\sin 37 = \underline{2.6\ Nm}$

- Pivot point - Some objects have a natural pivot point, but any point can be selected as a pivot point. The value of the torque will depend on the which point is the pivot. If there is no natural pivot point, it's best to choose:
  a) one of the ends b) a point where a force is applied

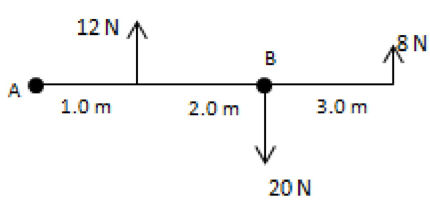


Rotational Equilibrium

Sum of clockwise torque = Sum of counterclockwise torque

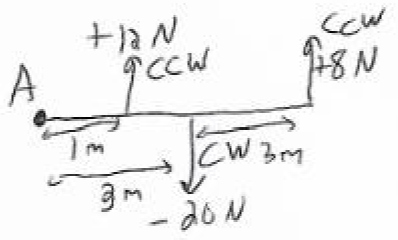
If the clockwise torque and counterclockwise torque are the same about a pivot point (any pivot point can be chosen), an object will not rotate about the pivot point and is in rotational equilibrium.

Example B: Check if the following object is in rotational and translational equilibrium:



$\Sigma F_x = \underline{0N}$ $\Sigma F_y = 12N - 20N + 8N = \underline{0N}$
∴ object is in translational equilibrium

Pivot at A



$$\sum \tau_A = (+12\,N)(1\,m) - (20\,N)(3\,m) + (8\,N)(6\,m)$$

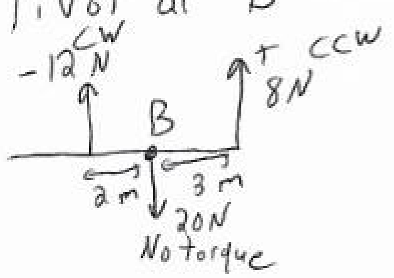
$$\sum \tau_A = 12\ Nm - 60\ Nm + 48\ Nm$$

$$\sum \tau_A = 0\ Nm$$

∴ object is in rotational equilibrium

A is a natural pivot as its on the left end. However, you don't have to pivot there. B is another possible point. The advantage of pivot at B is the 20 N force will apply no torque about this point since the distance from the force to point B is zero. Either way, the answer will be the same; there is no point that is necessarily the correct point to pivot. You just have to be consistent with the signs of the torque about that pivot and the distances to get correct answers.

Pivot at B



$$\sum \tau_B = (-12\,N)(2\,m) + (20\,N)(0\,m) + (8\,N)(3\,m)$$

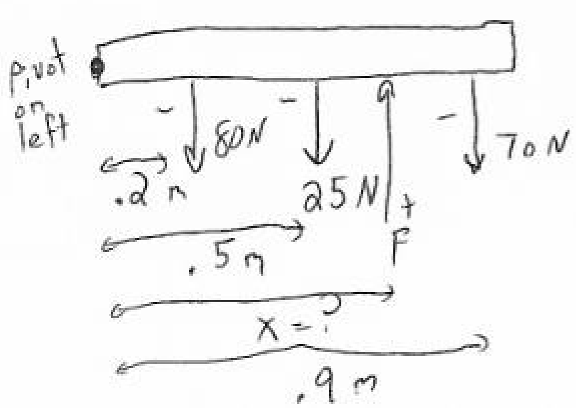
$$= -24\ Nm + 0 + 24\ Nm$$

$$\sum \tau_B = 0$$

Example C: A uniform meter stick weighs 25.0 N. An 80.0 N weight is hung at the 20.0 cm mark and a 70.0 N weight is hung at the 90.0 cm mark.

a) What is the magnitude of the upward force needed to balance the meter stick?

b) Where should the force be applied so that the stick hangs horizontally?



a) $\sum F_y = 0 = F - 80\,N - 25\,N - 70\,N$

$\underline{F = 175\,N}$

b) $\sum \tau_{Left} = 0$

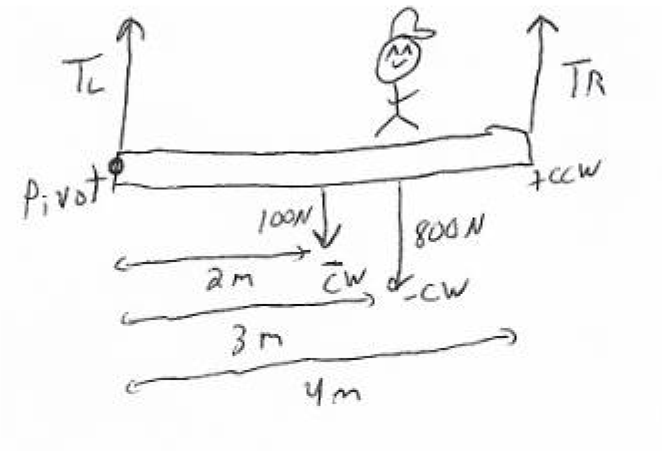
$\sum \tau_{Left} = (-80)(.2) - (25)(.5) + F(x) - (70).9$

$x = .52$

Example D: A 800 N painter stands 3.00 m from the left end of a scaffold that is 4.00 m long. The uniform scaffold weighs 100 N and is hung by a chain at each end. What is the tension in each chain?

Many problems involving static equilibrium will require working with both translational equilibrium and rotational equilibrium. Example C) only had one unknown force so it was easy to start using translational equilibrium.

This problem has two unknown forces, so it would not be possible to start using translation equilibrium. This problem can be solved by using rotational equilibrium first and pivoting at one of the ends of the scaffold. Pivoting at one of the ends means that there will be torque from this force, leaving only one unknown in the equation that can easily be solved for. For this example, it will be pivoted on the left.



$$\sum \tau_{left} = 0$$

$$\cancel{T_L(0)}^{0} - (100N)(2m) - (800N)(3m) + T_R\,4 = 0$$

$$-2600\,N + 4\,T_R = 0$$

$$T_R = 650\,N$$

This problem could be completed by now pivoting on the right to solve for the left tension, but it's easier to just use translational equilibrium since there will now only be one unknown force in the equation.

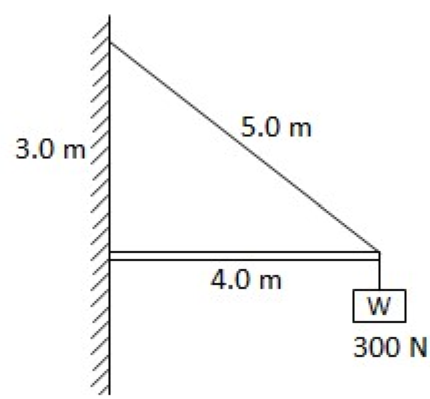
$$\sum F_y = 0$$

$$T_L + T_R - 100N - 800N = 0$$

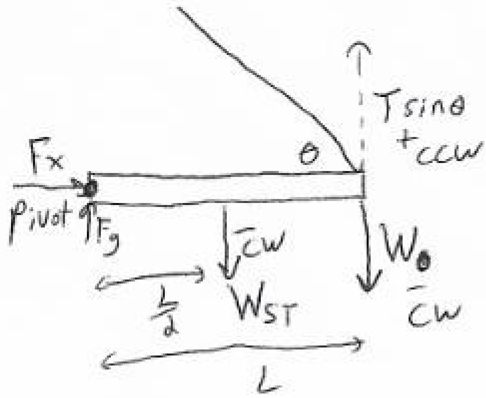
$$T_L + 650N - 900N = 0$$

$$T_L = 250\,N$$

Example E: The horizontal strut in the figure to the right is uniform and weights 100 N. Find a) the tension in the cable. b) the normal force of the wall on the strut.



For this problem, it's best to pivot on the left end. The length is not given, but will cancel out. Only the vertical component of the tension in the string applies torque about the pivot. There are unknown wall forces, so this problem must be pivoted at the lefthand side at the wall so only the forces right of the wall are in the equation.



$$\sum \tau_{pivot} = 0$$

$$-W_{ST}\left(\frac{\cancel{L}}{2}\right) - W(\cancel{L}) + T\sin\theta\,\cancel{L} = 0$$

$$-100N\left(\frac{1}{2}\right) - (300N) + T\frac{3}{5} = 0$$

$$\underline{T = 583\ N}$$

There are two wall forces. F_x, the horizontal component is due to the normal force of the wall pushing against the strut. The other wall force, F_y, is the vertical component which is friction against the strut sliding down the wall. These forces are the only unknown forces remaining in their respective directions so translational equilibrium can be used to solve for each.

<u>Wall forces</u>

$$\sum F_x = 0 = F_x - T\cos\theta \Rightarrow F_x = (583\ N)\left(\frac{4}{5}\right) = \underline{466\ N}$$

$$\sum F_y = 0 : F_y + T\sin\theta - W_{ST} - W$$

$$F_y + (583\ N)\left(\frac{3}{5}\right) - 100\ N - 300\ N = 0$$

$$\underline{F_y = +50\ N}$$

Rate your understanding: Torque

0	1	2	3	4
Torque? ¿Porque?	I understand the concept behind torque and can solve some problems.	I can set up and solve problems involving torque with minor errors.	I can set up and solve problems involving torque with no errors.	I can explain and teach the concepts behind torque.

6.3– Newton's 2nd Law for Rotation

Focus Question: What causes rotational motion?

In linear motion, acceleration is caused by force (Newton's second law). In rotary motion, angular acceleration is caused by torque (T).

Linear motion: $\sum F = ma$ Rotary motion: $\sum \tau = I\alpha$

Moment of inertia

*only valid for rigid bodies

Calculate momentum of inertia.

Only **mass** resists acceleration in linear motion. In angular motion, both the **mass** and **shape** of an object resist angular acceleration. A quantity called **moment of inertia** is a property of both mass and shape and is a measure of an object's ability to resist rotation.

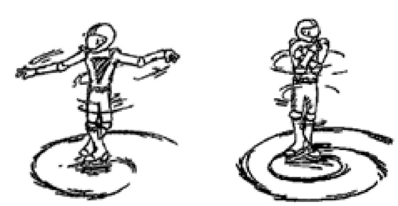
Rotational inertia is analogous to mass, but more complex. Rotational mass is based on the mass of a body and the distance of the mass from the axis of rotation.

Take 2 rods: Rod 1  Rod 2 

If a mass was placed on the end of each rod, and force was applied to one end, the mass at the end of rod 2 would cover a greater distance. . However, it would have a larger circumference to travel around, so the angular displacement would not be as large as for the smaller rod.

Formulas for rotational inertia are derived in calculus. In Physics 1, the formula will always be provided. However, you are expected to know qualitatively which objects have more inertia (more resistance to rotation).

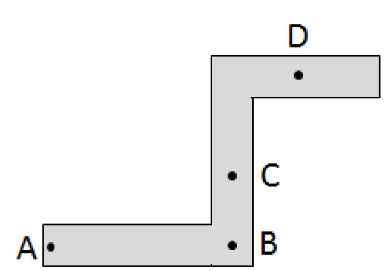
Example A: An Olympic ice skater is spinning and pulls her arms in to speed up her spin. Explain how this is possible with physics.



It's harder to make a given mass rotate around an axis that it's far from than one that it's close to.

When the skater pulls her arms in, she brings more of her mass closer to the center of axis of rotation, thus increasing her angular acceleration.

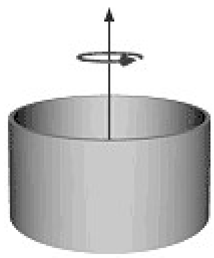
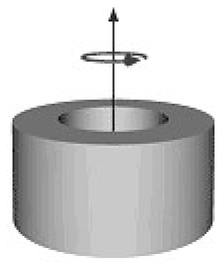
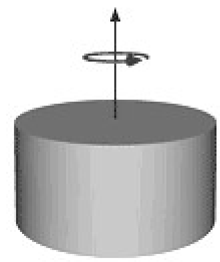
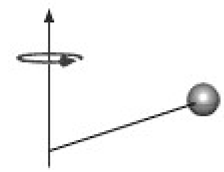
Example B: The thingamajic shown can be rotated around anyone of 4 pivot points shown. Rank the moment of inertia of the object about each pivot point.

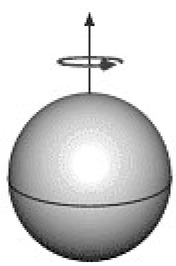
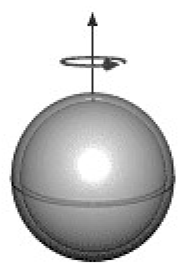
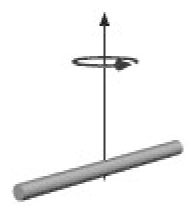
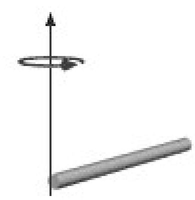


I is greater (I am greater?) when pivoted further from the center, which is around C's location.

$$I_A > I_D > I_B > I_C$$

Formulas for moment of inertia of common objects:

			
Thin ring of radius r and mass m	Ring of inner radius r and outer radius R.	Solid cylinder of radius r and mass m.	Point mass m at radius r.
$I = mr^2$	$I = m(R^2 + r^2)/2$	$I = (mr^2)/2$	$I = mr^2$

			
Solid sphere of mass m and radius r.	Hollow sphere of mass m and radius r.	Rod of length L and mass m hinged at center.	Rod of length L and mass m hinged at end.
$I = 2/5(mr^2)$	$I = 2/3(mr^2)$	$I = (mr^2)/12$	$I = (mr^2)/3$

*Units of moment of inertia: **kg$\cdot$ m^2**

Apply Newton's 2nd Law for rotation.

Example C: A solid spherical object with a mass of 5 kg and a radius of 50 cm experiences a force of 8 N applied at an angle of 40° with its axis of rotation. Find the angular acceleration of the sphere.

$$\tau = I\alpha$$

$$Fr\sin\theta = \frac{2}{5} m r^{\cancel{2}} \alpha$$

$$\alpha = \frac{5F\sin\theta}{2mr} = \frac{5(8\,N)\sin 40}{2(5\,kg)(.5\,m)} = 10.3\ 1/s^2$$

Example D: A tennis ball has a mass of .32 kg and a radius of .056 m. The rotational inertia of the ball is given by $I = \frac{2}{5}mr^2$, where r is the radius of the ball. What torque is required to give the ball an angular velocity of 5.0 rads/sec in .60 seconds?

$$\tau = I\alpha$$
$$\tau = \frac{2}{5}mr^2\alpha$$
$$\tau = \frac{2}{5}(.32\,kg)(.056\,m)^2(8.3\,\tfrac{1}{s^2})$$
$$\underline{\tau = .003\ Nm}$$

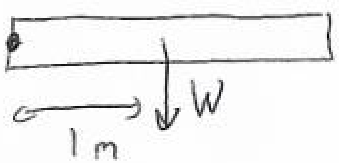
Kinematics to find α

$\omega_i = 0 \quad \omega_f = 5\,\tfrac{1}{s} \quad t = .6\,s$

$$\omega_f = \omega_i + \alpha t \Rightarrow \alpha = \frac{\omega_f}{t}$$
$$\alpha = \frac{5\,\tfrac{1}{s}}{.6\,s} = 8.3\,\tfrac{1}{s^2}$$

Example E: A rod is hinged on its left side. The uniform rod is 2.0 m long and has a mass of 3.6 kg. What is the angular acceleration of the rod at the instant is released from a horizontal position?

When the rod is released, its weight (which acts at the center of mass; the midpoint of the rod) causes applies a torque about the left side hinge. Diving this torque by the moment of inertia can be used to find the angular acceleration.



$$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I}$$
$$\alpha = \frac{-W(\frac{L}{2})}{\frac{1}{3}mL^2} = \frac{-(3.6\,kg)(10\,\tfrac{m}{s^2})(\frac{2\,m}{2})}{\frac{1}{3}(3.6\,kg)(2\,m)^2}$$
$$\alpha = -7.5\ \tfrac{rad}{s^2}$$

*Since the torque and acceleration are clockwise, these quantities are negative.

Rate your understanding: Newton's 2nd Law for Rotation

0	1	2	3	4
I do not know da wae.	I understand the concepts behind rotary motion and can solve some problems with help.	I can solve problems involving rotary motion with minor errors.	I can solve problems involving rotary motion with no errors.	I can explain and teach the concepts behind motion around an axis. Pretty righteous.

6.4– Conservation of Angular Momentum

Focus Question: What is angular momentum?

Define momentum in angular motion.

- Angular momentum is similar to linear momentum.
  Angular Momentum (**L**)
  L - a measure of how difficult it is to stop something from spinning:

$$L = I\omega$$

  *units – $kg \cdot m/s$

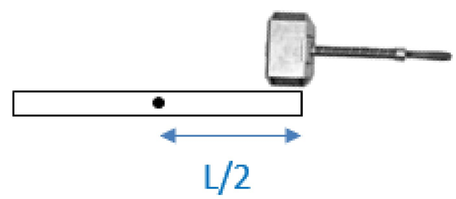
- Angular Impulse – An angular impulse is an outside torque (an outside force applied at distance) applied for a certain time. If an outside torque acts, the angular momentum of a system changes.

Angular Impulse = Change in angular momentum

$$J_\theta = \tau t$$

$$J_\theta = \Delta L$$

Example A: A uniform 3 kg rod is 2 m long and free to rotate about its center. By the hammer of Thor, the rod is struck by a 1500 N force for .08 s directly perpendicular to the rod. What is the angular velocity of the rod after impact? (I for rod pivoted at center = $\frac{1}{12}ML^2$)



$J_\theta = \Delta L$

$\tau t = I\omega_f - \cancel{I\omega_i}^{\,0}$ (starts at rest)

$-F\frac{L}{2}t = \frac{1}{12}ML^{\cancel{2}}\omega_f$

$\Rightarrow \omega_f = \frac{-12Ft}{2ML} = -120 \text{ rad/s}$

Apply conservation of angular momentum to solve problems.

In an angular "collision", angular momentum is always conserved unless external torques act on the system.

L is always conserved in an isolated system unless there is a net external torque.

$$L_i = L_f$$

*internal torques do not change angular momentum of a system (but can change the energy)

Example B: A baggage carousel has a mass of M=500 kg and a is in the shape of a disk with r = 2.0 m. It rotates at 1.0 rad/s when ten pieces of luggage with average mass of m = 20 kg are dropped on the carousel. Find the speed of the carousel after the luggage is added if no external torques act on the carousel.

Before the "collision", the baggage carousel spins at a rate of 1.0 rad/s. When the luggage is dropped on, the momentum of inertia of the carousel will increase, and its angular velocity will decrease as a result. Each piece of luggage will be treated as a point mass. The moment of inertia for a point mass is given by mr^2, where r is the distance to the center of rotation.

*The carousel is a solid disc with moment of inertia $\frac{1}{2}mr^2$.

$$L_i = L_f$$

multiply by 10 since there are 10 pieces

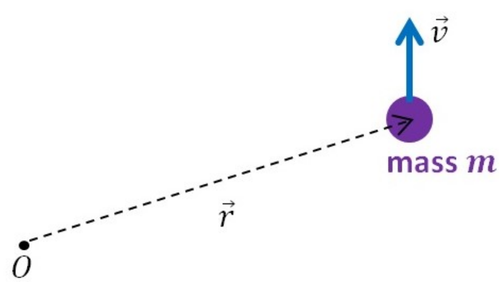
$$I_{carousel}\,\omega_i = (I_{carousel} + 10\, I_{Luggage})\,\omega_f$$

$$\frac{1}{2}Mr^2\omega_i = \left(\frac{1}{2}Mr^2 + 10mr^2\right)\omega_f$$

$$\Rightarrow \omega_f = \frac{\frac{1}{2}Mr^2\omega_i}{\frac{1}{2}Mr^2 + 10mr^2} = \frac{\frac{1}{2}(500\text{ kg})(2\text{ m})^2(1\,^1/_s)}{\frac{1}{2}(500\text{ kg})(2\text{ m})^2 + 10(20\text{ kg})(2\text{ m})^2}$$

$$\omega_f = .56\ \text{rad/s}$$

- The angular momentum, $\vec{L}$, of a particle about an axis perpendicular to the plane of the particles motion:



An object moving linearly through space can also have angular momentum about some reference point. For point mass moving linear, its angular moment a distance r away along a perpendicular axis is given:

$$L = mvr$$

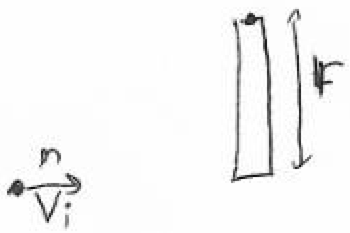
***In a collision with both linear and angular motion, linear and angular momentum are both conserved separately.**

Example C: A bullet of mass m = .20 kg is shot at a 40 kg door at its unhinged end. The speed of the bullet is initially 200 m/s and travel perpendicular to the plane of the door. The bullet becomes embedded in the door, which is 2.5 m high and 1.5 m wide.

a) What is the angular velocity of the door after the door after the bullet strikes?

*The door is a rod hinged at one end ($I=\frac{1}{3}ML^2$). The height is negligible.

Before collision



After collision

$$L_i = L_f$$

$$m V_i r = (I_{door} + I_{bullet})\omega$$

$$\omega = \frac{m v_i r}{\frac{1}{3} M r^2 + m r^2} = \frac{(.2\,kg)(200\,m/s)(1.5\,m)}{\frac{1}{3}(40\,kg)(1.5m)^2 + (.2\,kg)(1.5m)}$$

$$\omega = 1.97\ rad/s$$

b) How would the answer change if the bullet hit closer to the door?

The angular velocity will **increase.** Although there will be more less momentum (mvr), there will also be significantly less moment of inertia (mr^2) after the collision. Looking at the equations, moment of inertia will decrease far more than initial moment will decrease if the bullet hits closer to the door. Therefore the door will rotate faster with a collision closer to its hinge due to having significantly less moment of inertia in this case.

Rate your understanding: Conservation of Angular Momentum

0	1	2	3	4
The classroom clock looks like it has hardly any angular momentum.	I understand some concepts in angular momentum.	I apply conservation of angular momentum with only a few errors.	I apply conservation of angular momentum with no errors.	I can explain and teach the concepts angular momentum.

6.5– Work & Energy in Rotational Motion

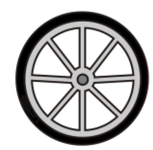
Focus Question: How do we deal with a system with translational and rotational motion?

Solve for rotational kinetic energy.

Kinetic energy of rotation – Energy an object possess due to its rotational motion. $K = \frac{1}{2}I\omega^2$

Analyze systems with moving axis of rotation.

- Often times, a body has both translational and rotational motion, such as the wheel on a car.
- Newton's Laws for rotational mechanics still remain valid as long as:
  - The axis of rotation passes through the center of mass.
  - rotation does not change direction.
- Total kinetic energy of a rigid body – A rolling object has both linear kinetic and rotational kinetic energy.



$$K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

Example A: A soccer ball of mass M and radius R is rolling at a speed V. Find the total kinetic energy. The moment of inertia for a solid sphere is $\frac{2}{5}MR^2$.

$$K = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MV^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2$$

$$K = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \underline{\frac{7}{10}Mv^2}$$

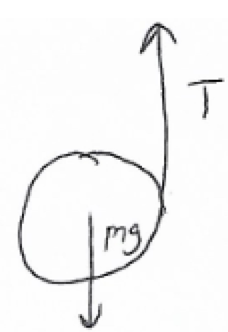
*Total kinetic energy does *not* depend on radius*

Example B: A yo-yo is a solid disk or radius R and mass M. The yo-yo is released from rest while the free end of the yo-yo string is held stationary. Find a) the acceleration of the yo-yo and b) the tension in the yo-yo string.

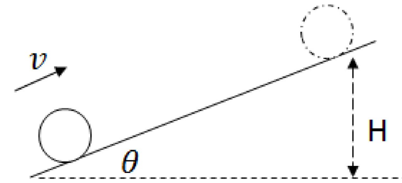
In problems involving linear and rotational motion, both F=ma and τ=Iα need to be used. For this problem, tension supplies a force and a torque. Since mg acts at the center of the yo-yo, it only applies a force. In both cases, the direction of motion (CCW and downwards) is to be taken as positive. As a result, tension is negative as a force, but is a positive torque.

$$\Sigma F = Ma = Mg - T$$
$$Ma = Mg - Ma$$
$$2a = g$$
$$\underline{a = g/2}$$

$$\Sigma \tau = I\alpha$$
$$TR = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$
$$T = \frac{1}{2}Ma$$
$$T = \frac{1}{2}M\left(\frac{g}{2}\right) = \underline{\frac{Mg}{4}}$$



Example C: A solid wheel of mass M and radius R moves up an incline as shown. The speed of the ring is v as it enters the incline.



Background When friction is present, it is used to convert rotational kinetic energy into linear kinetic energy. The magnitude of the negative rotational work done by friction equals the positive linear work done by friction. Thus friction does no net-work.

a) Find how high up the incline the wheel goes if: the wheel slips while going up the incline. (frictionless)

If there is no friction, the rotational motion of the wheel will not be converted to linear motion. This problem works the same as sliding.

$$\cancel{U_i} + K_i = U_f + \cancel{K_f}$$
$$\frac{1}{2}Mv^2 = MgH \Rightarrow H = \sqrt{2gH}$$

b) Find how high up the incline the wheel goes there is no slipping of the wheel as it moves up the incline. (friction is present)

If there is friction, the rotational of the wheel helps it move forward and it can convert its rotational kinetic energy into potential energy.

$$\cancel{U_i} + K_i = U_f + \cancel{K_f}$$
$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = MgH$$
$$\frac{1}{2}\cancel{M}v^2 + \frac{1}{2}\left(\frac{1}{2}\cancel{M}\cancel{R^2}\right)\left(\frac{v}{\cancel{R}}\right)^2 = \cancel{M}gH$$
$$\frac{1}{2}v^2 + \frac{1}{4}v^2 = gH$$
$$\frac{3}{4}v^2 = gh \Rightarrow \underline{h = \frac{3v^2}{4g}}$$

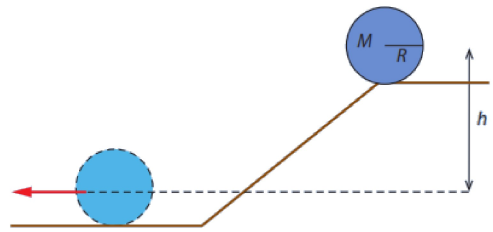
In a), only the linear kinetic energy is converted to potential energy. In b), both linear and rotational kinetic energy are converted into gravitational potential energy, which is why the answer to b) is greater.

c) How would the answer to b) change if the wheel were solid?

In this case, there would be more initial energy (more I at the beginning), so the wheel will go higher.

NOTE In the no friction case (slipping), only translational kinetic energy is converted into gravitational potential energy. In the case with friction (no slip), more kinetic energy is converted into gravitational potential energy and the wheel will rise more. The torque causes the object to slow down is provided by the parallel component of weight. Since friction causes the angular acceleration to be less negative tan the slipping case, **friction points in the positive direction** for this problem.

Example D; Rolling down an incline: A wheel of radius M and radius R rolls without slipping down an incline.



a) Find the linear speed of the center of mass of the wheel when it reaches the bottom of the incline.
b) Calculate the acceleration of the wheel down the incline.
c) Determine the minimum coefficient of friction required to prevent slipping down the incline.

a) Using conservation of energy:

$$U_i + \cancel{K_i} = \cancel{U_f} + K_f$$
$$MgH = \frac{1}{2} Mv^2 + \frac{1}{2}\left(\frac{1}{2} M\cancel{R^2}\right)\left(\frac{v}{\cancel{R}}\right)^2$$
$$\cancel{M}gH = \frac{1}{2} \cancel{M}v^2 + \frac{1}{4}\cancel{M}v^2$$
$$gH = \frac{3}{4} v^2 \Rightarrow \underline{v = \sqrt{\frac{4gH}{3}}}$$

b) The force of friction applies a positive torque since it causes the wheel to rotate downhill, but a negative force since it resists the sliding motion downhill. *friction always is to be taken as acting uphill on an incline. Gravity (the parallel component) is a positive force since it propels the wheel downhill. Gravity causes no torque since it acts at the center of mass. The normal force applies no torque or force since it is parallel to the axis of rotation (no torque) and has no component in the direction of motion (no force).

$$\Sigma F_{\parallel} = Ma = Mg\sin\theta - F_f$$
$$\cancel{M}a = \cancel{M}g\sin\theta - \frac{1}{2}\cancel{M}a$$
$$\underline{a = \frac{2}{3} g\sin\theta}$$

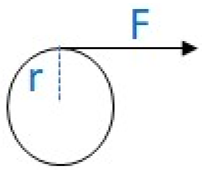
$$\Sigma \tau = I\alpha$$
$$F_f \cancel{R} = \frac{1}{2} MR^2\left(\frac{a}{\cancel{R}}\right)^2$$
$$F_f = \frac{1}{2} Ma$$

c) To prevent slipping, there are two conditions: $v = r\omega$ and $\mu = mg\cos\theta$.

$$F_f = \mu Mg\cos\theta = \frac{1}{2} Ma$$
$$\mu \cancel{Mg}\cos\theta = \frac{1}{2}\cancel{M}\frac{2}{3}\cancel{g}\sin\theta$$
$$\mu = \frac{1}{3}\tan\theta$$

Solve for work and power in rotational motion.

A force applied to a rotating body does work on the body. The work can be expressed with torque and angular displacement:



$$W = \Delta K \rightarrow \tau\theta = \Delta L$$

$$\rightarrow Fr\theta = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

*only the component of the force perpendicular to the axis of rotation does work.

Example D: A 12.0 kg golf cart wheel has a radius of .30 m. The wheel is brought to rest from an initial speed of 30 rad/s. The wheel makes 10 revolutions while slowing down.

a) What is the work done to slow the wheel?
b) What torque is required to slow the wheel?
c) What is the power required to stop the cart?

Since the system stops and loses energy, the work, torque, and power will all be negative.

a) $W = \Delta K = \cancel{\frac{1}{2} I \omega_f^2}^{\,0} - \frac{1}{2} I \omega_i^2 = -\frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_i^2$

$W = -\frac{1}{2}\left(\frac{1}{2}\right)(12\ kg)(.3\ m)^2(30\ \tfrac{1}{s})^2 = \underline{-243\ J}$

b) $W = \tau\theta \Rightarrow \tau = \frac{W}{\theta} = \frac{-243\ J}{20\pi} = \underline{-3.9\ Nm}$

c) $P = \frac{W}{t}$

$P = \frac{-243\ J}{4.2\ s} = \underline{-58\ W}$

Kinematics to find t

$\theta = 20\pi$

$\omega_i = 30\ \tfrac{1}{s}$

$\omega_f = 0\ \tfrac{1}{s}$

$t = ?$

$\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$

$t = \frac{2\theta}{\omega_i} = \frac{2(20\pi)}{30} = 4.2\ s$

Rate your understanding: Rotational Motion

0	1	2	3	4
My physics grade is rolling in its grave.	I understand the concepts behind rotary motion and can solve some problems with help.	I can solve problems involving rotary motion with minor errors.	I can solve problems involving rotary motion with no errors.	I can explain and teach the concepts behind motion around an axis.