AP Physics - Unit 7

Tipper #3 - Unit 7 - KEY



#1

*Answer: E = F > B = D > C > A.*

*Ranked on v since KE is proportional to ω squared or v squared and the mass is constant.*

A Rolling Body has Linear KE along w/ Rotational KE

$$KE_{Total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$I = mR$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mR\omega^2$$

Same mass

we know that in Rolling w/out Slipping $V = r \times \omega$

$\therefore$ $KE_{Total} = \frac{1}{2}\cancel{m}v^2$

All Depends on V

E = F > B = D > C > A

60 60 50 50 40 30

*Answer: A = C > B > D*

#2

*Each of these objects begins with gravitational potential energy at the top of the ramp that is comverted to kinetic energy at the bottom. The objects will have both translational and rotational KE at the bottom. Energy is conserved so we can set mgh = 1/2mv² + 1/2Iω². Assuming that all of the objects roll without slipping the translational KE will be proportional to the rotational KE. After some algebra we find that the masses cancel for each case, since mgh = final kinetic energy which also involves the mass. That means the determining factor is the fraction in the moment of inertia for each object, since that determines which object will have more translational kinetic energy, and consequently will get down the incline faster.*

A&C Solid Sphere

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$I = \frac{2}{5}mR^2$

$\omega = \frac{V}{R}$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\frac{2}{5}\right)\cancel{m}\cancel{R^2}\left(\frac{V}{\cancel{R}}\right)^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{5}\frac{2}{10}v^2$$

$\frac{5}{10}$ $\frac{2}{10}$

$$gh = \frac{7}{10}v^2$$

$$v^2 = \frac{10}{7}gh$$

A = C > B > D

hollow Sphere B

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$I = \frac{2}{3}MR^2$

$\omega = v/R$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\frac{2}{3}\right)\cancel{m}\cancel{R}(v/\cancel{R})^2$$

$$gh = \frac{1}{2}v^2 + \frac{3}{6}v^2$$

$\frac{3}{6}$

$$gh = v^2$$

$$v^2 = gh$$

Hoop D

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$I = MR^2$

$\omega = v/R$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\cancel{m}\cancel{R^2}\left(\frac{v}{\cancel{R}}\right)^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{2}v^2$$

$$v^2 = gh$$

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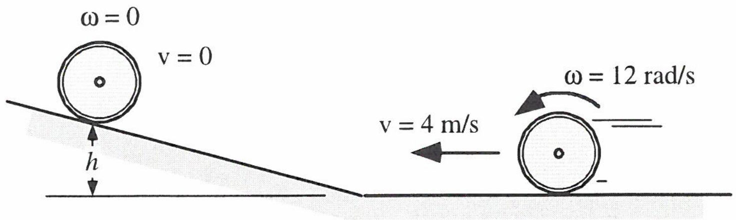
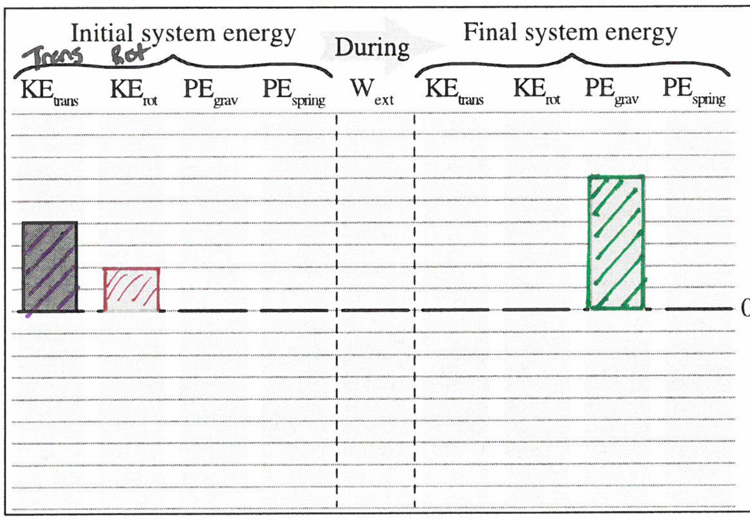
#3

**B6-BCT28: SOLID DISK ROLLING UP A RAMP—ROTATIONAL ENERGY BAR CHART**

A solid disk is initially rolling without slipping along a flat, level surface. It then rolls up an incline, coming momentarily to rest as shown.




**Complete the qualitative energy bar chart below for the earth-disk system for the time between when the disk is rolling on the horizontal and when it has rolled up the ramp and is momentarily at rest.** Put the zero point for the gravitational potential energy at the height of the center of the hoop when it is rolling on the horizontal surface.




| Bar chart key | |
|---|---|
| $KE_{trans}$ | Translational kinetic energy |
| $KE_{rot}$ | Rotational kinetic energy |
| $PE_{grav}$ | Gravitational potential energy |
| $PE_{spring}$ | Spring potential energy |
| $W_{ext}$ | Work done by external forces |

Use $g = 10\ m/s^2$ for simplicity

**Explain your reasoning.**

*Answer: Since the disk is initially at the zero gravitational potential energy height, there is no initial potential energy. The disk is translating (its center of mass is moving) and rotating, so there are initial kinetic energies of translation and rotation. For a disk rolling without slipping, $I = (0.5)mr^2$ and $\omega = v/r$, so $KE_{rot} = (0.5)I\omega^2 = (0.5)((0.5)mr^2)(v/r)^2 = (0.25)mv^2$. The initial rotational kinetic energy is therefore one-half of the initial translational kinetic energy. In the final state, the hoop is at rest (momentarily) on the incline, so it has neither rotational nor translational kinetic energies. All of the initial system energy has been converted into gravitational potential energy.*

Rot

$KE_{Rot} = \frac{1}{2} I \omega^2$

$\omega = V/R$

$I = \frac{1}{2} mR^2$ (Disk)

$= \frac{1}{4} m\cancel{R^2} \left(\frac{V}{\cancel{R}}\right)^2$

$KE_{Rot} = \frac{1}{4} mV^2$

$\frac{1}{2} \frac{1}{2}$

Trans

$KE_T = \frac{1}{2} mv^2$

$\therefore KE_{Rot}$ is $\frac{1}{2} KE_T$

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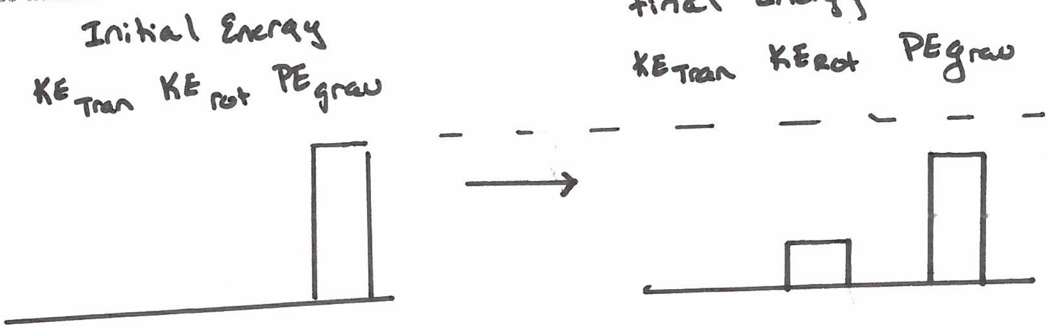
#4 *Answer: (i) The moment of inertia will be greater about the left end.*

*The moment of inertia is a measure of how difficult it is to change the rate of rotation of an object. When mass is close to the axis of rotation, it is not as difficult to speed up or slow down in rotation. The moment of inertia is greatest when more mass is further from the axis of rotation.*

*Answer: It will be below point A.*

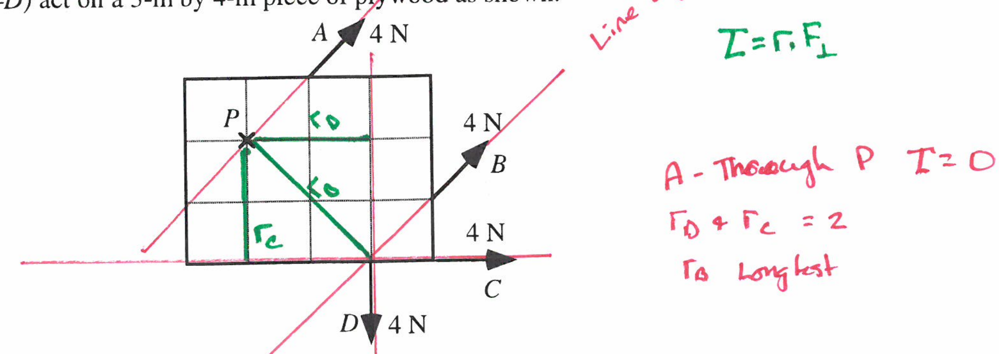
#5 *The initial potential energy will be converted into both rotational kinetic energy and translational kinetic energy at point B. After the ball is launched, it will still be spinning. Once it is in the air there is no external torque acting on it so it will continue to rotate at the angular velocity with which it was launched. At the top of its trajectory, it will still have rotational kinetic energy, so the final gravitational potential energy will be less than the initial.*




#6

**B6-RT25: Four Forces Acting on a Piece of Plywood—Torque**

Four 4-Newton forces (*A–D*) act on a 3-m by 4-m piece of plywood as shown.




**Rank the magnitudes of the torques due to the four forces about point *P*.**

| 1 Greatest | 2 | 3 | 4 Least | OR | All the same | All zero | Cannot determine |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

**Explain your reasoning.**

*Answer: B > C = D > A.*

*The torque due to each force is the product of the force (which is the same in all cases) and the perpendicular distance between the line of action of the force and the pivot point. Since the line of action for force A acts through point P, this force exerts no torque about point P. The perpendicular distance is 2 meters in cases C and D, and 2.82 meters in case B. The ranking of the torques will be the same as the ranking of the perpendicular distances.*

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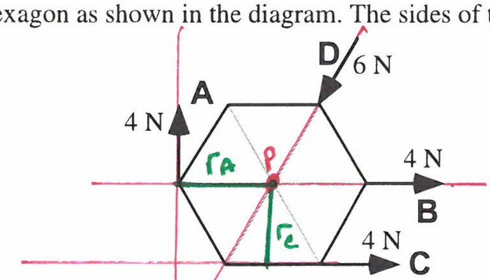
#7

**B6-RT12: FOUR FORCES ACTING ON A HEXAGON—TORQUE ABOUT CENTER**

Four forces act on a plywood hexagon as shown in the diagram. The sides of the hexagon each have a length of 1 m.




— Line of Action

$F_D$ + $F_B$ Throu pivot ∴ $\tau = 0$

$r_A > r_c$ ($\perp$ R), $F_A = F_c$

$\tau = r_\perp F$

∴ $\tau_A > \tau_A$

**Rank the magnitude of the torque applied about the center of the hexagon by each force.**

| 1 Greatest | 2 | 3 | 4 Least | OR | All the same | All zero | Cannot determine |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

**Explain your reasoning.**

*Answer: A > C > B = D.*

*The magnitude of the torque due to each force is equal to the magnitude of the force times the perpendicular distance between the line of action of that force and the pivot point. The lines of action of forces B and D pass through the center of the hexagon, so the torques due to forces B and D are both zero. The perpendicular distance between the line of action and the pivot point is equal to the height of one of the triangles shown for force C, and is equal to the side of a triangle for force A. The side of the triangle is longer than the height, so the torque due to force A is greatest.*

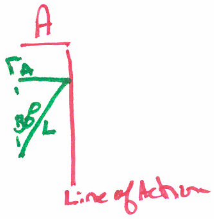
*Answer: B > D > C > A.*

#8

*The torque depends on the weight of the sign (mg) times the distance from the line of action of this weight to the point of attachment. The line of action of the weight is along the rope that is suspending the sign, which is one rod length (L) away from the attachment point in cases B and D, closer than that in case C, and closer yet (0.5L) in case A. Since A and C have the smallest masses and C has the smallest distance, A will be larger than C, which will be the smallest. Since B and D have the largest masses and the largest distances, they will have the largest torques, with B > D.*

$\tau = r_\perp F$

A



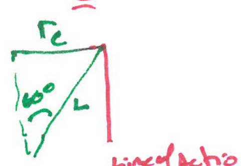

$\tau = L \sin 30 F$

$(.5L)\ 50$

B

$\tau = L\ 100kg$

C




$\tau = L \sin 60 F$

$\tau = (.86L)\ 50$

D

$\tau = L\ 90kg$

∴ B > D > C > A

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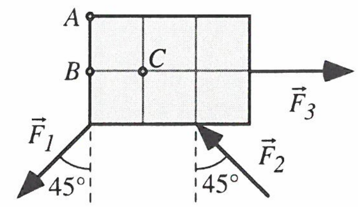
## B6-QRT09: THREE FORCES APPLIED TO A RECTANGLE—TORQUE DIRECTION

Three forces of equal magnitude are applied to a 3-m by 2-m rectangle. Forces $\vec{F}_1$ and $\vec{F}_2$ act at 45° angles to the vertical as shown, while $\vec{F}_3$ acts horizontally.




**(a) Is the torque by $\vec{F}_1$ about point *A* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** _____

**Explain your reasoning.**

*Answer: Clockwise The component of $F_1$ that is horizontal would pull the bottom of the rectangle to the left.*

**(b) Is the torque by $\vec{F}_1$ about point *B* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** _____

**Explain your reasoning.**

*Answer: Clockwise. The situation is the same as for A except that the moment arm is shorter.*

**(c) Is the torque by $\vec{F}_1$ about point *C* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** _____

**Explain your reasoning.**

*Answer: Zero. The line of action of the force goes through C so there is a zero moment arm.*

**(d) Is the torque by $\vec{F}_2$ about point *A* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** _____

**Explain your reasoning.**

*Answer: Zero. The line of action of the force goes through A.*

**(e) Is the torque by $\vec{F}_2$ about point *B* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** _____

**Explain your reasoning.**

*Answer: Counterclockwise. $F_2$ will push the rectangle up making it rotate CCW about B.*

**(f) Is the torque by $\vec{F}_2$ about point *C* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** _____

**Explain your reasoning.**

*Answer: Zero. The line of action of the force goes through C.*

**(g) Is the torque by $\vec{F}_3$ about point *A* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** _____

**Explain your reasoning.**

*Answer: Counterclockwise. $F_3$ will pull the rectangle up around A.*

**(h) Is the torque by $\vec{F}_3$ about point *B* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** _____

**Explain your reasoning.**

*Answer: Zero. The line of action of the force goes through B.*

**(i) Is the torque by $\vec{F}_3$ about point *C* (i) *clockwise*, (ii) *counterclockwise*, or (iii) *zero*?** _____

**Explain your reasoning.**

*Answer: Zero. The line of action of the force goes through C.*

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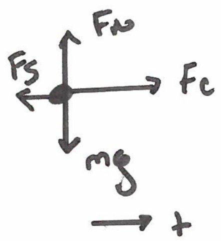
#10 $A = B > D > C$




$\Sigma F_x = F_c - F_s = m\cancel{a}^{\,0}$

$F_s = F_c$

$F_c = \dfrac{mv^2}{r}$

$v = r\omega$

$= \dfrac{m(r\omega)^2}{r}$

$F_c = mr\omega$

$F_s = mr\omega$    $\omega$ = constant in All CASES

$\therefore$ $F_s$ is dependent on $r$ & $m$

$F_c = mr\omega$

**A**

$F_s = (6)(2)$
$= 12$

**B**

$F_s = (4)(3)$
$= 12$

**C**

$F_s = (1)(5)$
$= 5$

**D**

$F_s = (2)(5)$
$= 10$

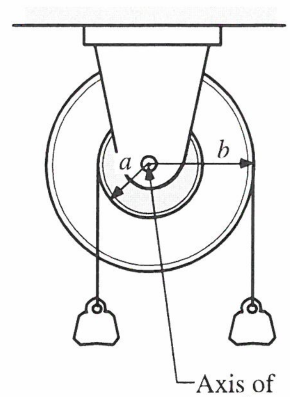
$\therefore$ $A = B > D > C$

#11

The Students answer & reasoning are Both Correct

## B6-QRT05: Pulleys with Different Radii—Rotation and Torque

A wheel is composed of two pulleys with different radii (labeled $a$ and $b$) that are attached to one another so that they rotate together. Each pulley has a string wrapped around it with a weight hanging from it as shown. The pulleys rotate about a horizontal axis at the center. When the wheel is released it is found to have an angular acceleration that is directed out of the page or counterclockwise.




**(a) Is the wheel going to rotate (i) *clockwise,* (ii) *counterclockwise,* or (iii) *none*?**

______

**Explain your reasoning.**

*Answer: The wheel is rotating counterclockwise, as application of the right-hand rule shows.*

**(b) Is the direction of the net torque on the pulley wheels (i) *clockwise*, (ii) *counterclockwise*, or (iii) *none*?** _____

**How do you know?**

*Answer: The net torque is directed out of the page since the net torque must be in the same direction as the angular acceleration.*

**(c) How do the masses of the two weights compare?**

**Explain your reasoning.**

*Answer: Since the angular acceleration is out of the page, the net torque is out of the page. The tension in the string attached to the smaller pulley is producing a torque that is out of the page, and the tension in the string attached to the larger pulley is producing a torque that is into the page. Since the net torque is out of the page, the torque due to the tension in the string on the left is greater than the torque due to the tension in the string on the right. Therefore tension in the string on the left times the radius a must be greater than the tension in the string on the right times b. Since a is smaller than b, it must be true that the tension in the string on the left is*