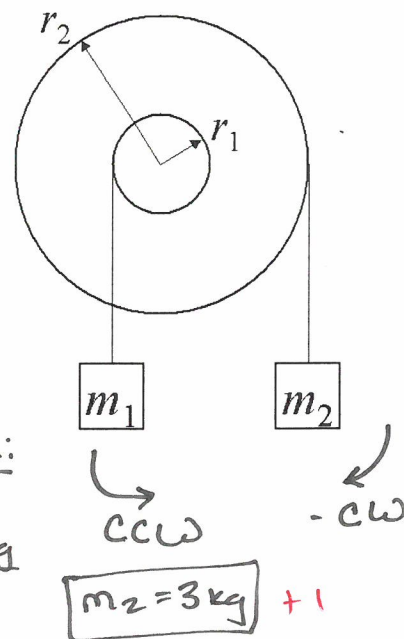


AP Physics Unit 7 – Torque and Rotational Motion
 Wkst – Rotational Motion – 3 FRQ

1.

A double-pulley is built by welding together a solid disk of radius r_1 to a larger disk of radius r_2 so that the disks are coaxial. The double-pulley is supported by a horizontal axis that passes through the plane of the page. One string is wound several times around the smaller disk and a block of mass m_1 is hung from the free end of the string to the left of the disk. Another string is wound several times around the larger disk and a block of mass m_2 is hung from the free end of the string to the right of the disk.



- (a) Suppose $m_1 = 9 \text{ kg}$ and $r_2 = 3r_1$. What mass must m_2 have in order for the system to remain at rest if the system were released from rest? Explain how you formulated your answer.

$\sum \tau_{\text{net}} = \tau_{\text{ccw}} - \tau_{\text{cw}} = 0$ (0 = Rest) ?
 $\tau_{\text{ccw}} = \tau_{\text{cw}}$
 $r_1 m_1 g = r_2 m_2 g$
 $r_1 m_1 = r_2 m_2$
 $m_2 = \frac{r_1 m_1}{r_2}$
 $r_2 = 3r_1$

$m_2 = \frac{r_1 m_1}{3r_1}$
 $m_2 = \frac{9 \text{ kg}}{3}$

$m_2 = 3 \text{ kg}$ +1

For parts (b) and (c), assume $m_1 = m_2$. The system is released from rest.

- (b) Which block rises upward once the system is released? Explain your reasoning.

$\uparrow ?$
 +1 Block 1 will rise upward

+1 Block 2 exerts more Torque and \therefore falls
 $\hookrightarrow \tau = mgr$ mass 2 has longer R

- (c) Which block has a greater magnitude of acceleration after the system is released, or do both blocks have the same magnitude of acceleration? Explain your reasoning.

+1 Block 2 has greater magnitude of acceleration

+1 Reason: Block 2 has a greater linear motion because it is connected to the larger radius wheel

- (d) Does the gravitational potential energy of the m_1 - m_2 -pulley-Earth system increase, decrease, or remain constant as the blocks accelerate after the system is released? Explain your reasoning.

+1 gravitational potential energy decrease. When release, motion occurs, so kinetic energy of system increase (was 0), Total Energy must remain constant $\therefore U_g \downarrow$

- (e) Is the linear momentum of the m_1 - m_2 system conserved during the interval after the system is released? Why or why not? NO

Reason 1 - unbalanced external force on the system

Reason 2 - The 2 blocks move in opposite directions w/ unequal speeds, so their momentums cannot cancel to zero, which was initial momentum

Block 2 and its string is removed. Block 1 is released with the system at rest at time $t = 0$ and allowed to fall a distance d while the double-pulley rotationally accelerates. The time t required for the block to fall the distance d is recorded. The values of m_1 , r_1 , and r_2 are also measured using appropriate equipment.

- (f) Explain how the measurements of d , t , m_1 , r_1 , and r_2 can be used to calculate the rotational inertia of the double-pulley. Your explanation will primarily consist of descriptive sentences but will necessarily require you to cite specific equations that model the situation and explain how the equations are used.

50's
The goal is to solve for I (Rotational Inertia) using measurements of d , t , r_1 , & r_2



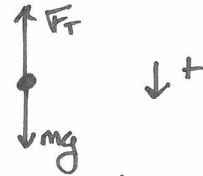
Inertia is the Net Torque \div By angular acceleration

$$I = \frac{\sum \tau_{\text{net}}}{\alpha}$$

The net torque is net forces acting on the pulley

$$\sum F = mg - F_T = ma$$

+1 $F_T = mg - ma$ The forces acting on the block relates to the block's acceleration



+1 $\therefore I = \frac{r_1 F_T}{\alpha}$

+1 $\alpha = \frac{a}{r}$ Relates the Angular Acceleration to linear acceleration

To find a

$$d = d_0 + v_0 t + \frac{1}{2} a t^2$$

+1 $a = \frac{2d}{t^2}$

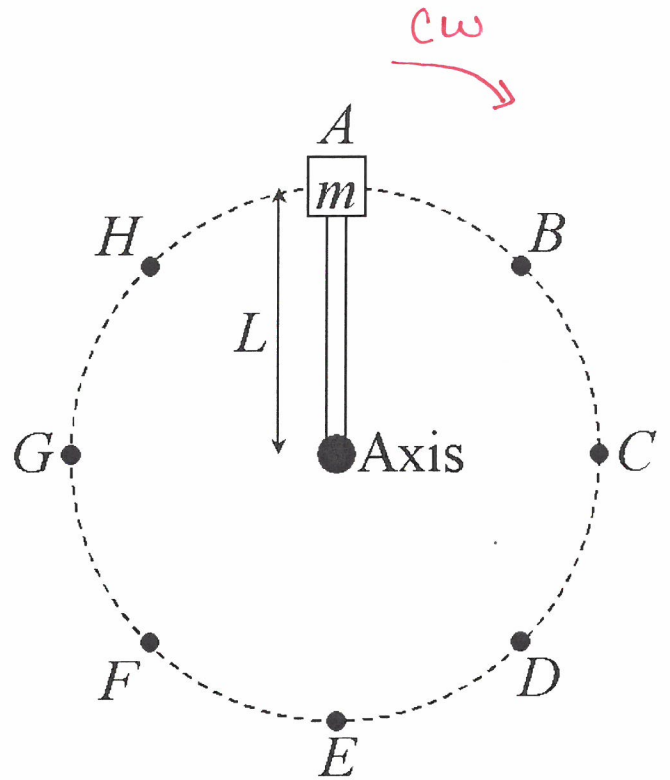
+1 So, solve for a , then determine α , solve for F_T . Then you have all variables for $I = \frac{r F_T}{\alpha}$

- (g) How would the value of the rotational inertia of the pulley be different if Block 1 were removed and Block 2 were instead used in the experiment? Explain your reasoning.

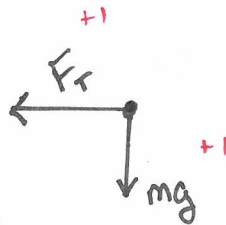
+1 Rotational Inertia will not change, because none of the properties of the pulley wheel were changed.

2. Rotational Dynamics

A dense, heavy block of mass m is connected to one end of a strong, thin, extremely light rod. The other end of the rod is fixed to a horizontal axis so that the center of the block is a distance L from the center of the axis. The assembly is oriented so that the rod is vertical with the block on top (at point A) and released. Because the system was not perfectly balanced at the instant it was released, the rod-block system rotates clockwise so that the block passes through points B , C , and D before reaching point E where the rod is again vertical, this time with the block at the bottom. The system continues rotating through points F , G , and H before coming momentarily to rest again at a point very close to and to the left of point A .



- (a) On the dot below, draw and label the forces (not components) acting on the block when it passes through point C .



- (b) Let a be the tangential acceleration of the block, the rate at which the speed is changing with time. At which point(s) is the block located when the magnitude of a is maximized? If there is a "tie" between two points where the magnitude of a is maximized, state both points. In any case, justify your choice(s).

+1 The max a of the Block will occur at points C & G . $a = \frac{\tau_{net}}{I}$

+1 The Torque is maximized at points C & G , Because the force & radius vector are perpendicular at those points



- (c) At which point(s) is the block located when the magnitude of the block's linear speed maximized? If there is a "tie" between two points where speed is maximized, state both points. In any case, justify your choice(s).

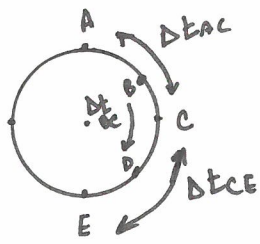
+1 The maximum linear speed will occur at point E

3 Possible Answers, need 1

The Torque ceases to be with the direction of acceleration & is about to become "against" the direction of accel

+1 or The Torque is zero at point E & zero Torque means angular speed is max
or Point E is where potential energy has decreased most \therefore KE is MAX

- (d) Let Δt_{AC} represent the time it takes for the block to go from point A to point C, Δt_{BD} the time to go from point B to point D, and Δt_{CE} the time to go from point C to point E. Rank these three time intervals and justify your ranking.



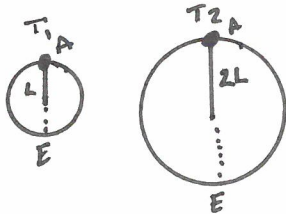
- +1 The Block speeds up the entire time from A to E
- +1 \therefore Speeding up Results in Shorter Time Intervals

Ranking Time intervals

+1 $\Delta t_{AC} > \Delta t_{BC} > \Delta t_{CE}$

The rod is replaced by a new rod that is longer so that the length L is doubled. The mass is released again from above the axis and rotates through the point directly below the axis. In this new situation, the time it takes for the block to move from above the axis to below the axis is T_2 . In the previous situation, the time it took for the block to move from point A to point E was T_1 . Angela believes that $T_2 > T_1$, while Blake believes that $T_2 < T_1$.

- (e) Give a plausible reason why Angela could be correct. Be sure to cite how specific physical quantities are different in the new situation and discuss their relationships with other physical quantities.



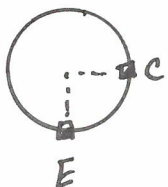
- $T_2 > T_1$
- +2 • The distance that the Block must travel is greater
or \therefore time is greater
 - +2 • Rotational Inertia of the System is greater
Because the Block is further from Axis

- (f) Give a plausible reason why Blake could be correct. Be sure to cite how specific physical quantities are different in the new situation and discuss their relationships with other physical quantities.

$T_2 < T_1$

- +1 • The greater U_g transform into \uparrow KE
 $\approx 2L$ vs L
this greater Energy transfer from U_g to KE Results in
+1 \uparrow Velocity \therefore Less time
- OR • The Torque is Greater $\tau = rF$
 $\approx 2L$ vs L Because
the Radius is greater

- (g) At which location is the block, point C or point E, when the magnitude of the force that the rod exerts on the block is maximized, or does the rod exert the same-magnitude force on the block at both points? Justify your answer.



max force Rod exerts on Block?

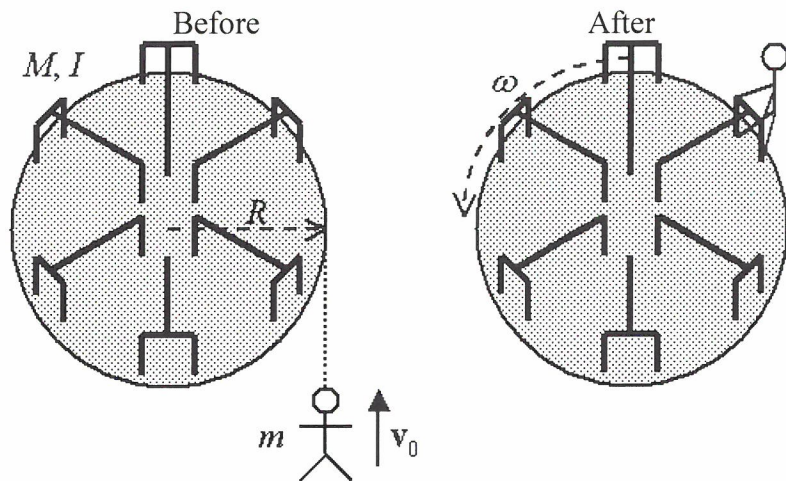
Point E, The force ~~on~~ due to the Rod is Centripetal force Acting on the Block.

Greater Centripetal force is greater when Block's speed is greater.



3. Rotational Dynamics

A merry-go-round on a playground consists of a solid circular-disk platform supported by a frictionless axis at its center. The platform is fitted with handles on which children can grab and hang on as the platform rotates. The merry-go-round has mass M , radius R , and rotational inertia I . The merry-go-round is at rest when a child of mass m runs with speed v_0 toward the merry-go-round, tangent to its edge. Upon reaching the edge, the child jumps on and hangs on to the merry-go-round, causing the child-merry-go-round system to have a constant angular speed ω , as shown in the diagram.



- (a) Is the linear speed of the child faster, slower, or the same after grabbing onto the merry-go-round when compared to the child's linear speed before? Justify your answer.

+1 The child moves slower.

+1 The merry-go-round exerts a backward force on the student as a reaction to the student's forward force on the merry-go-round.
* Collision - momentum is conserved! $(M+m)v = v_0M + v_0m$

OR
+1 Angular Momentum (L) is conserved, so merry-go-round speeding up (from rest) its rotation, requires the student's to slow down.

- (b) Is the magnitude of the angular momentum of the child-merry-go-round system after the child grabs onto the merry-go-round greater than, less than, or the same as the angular momentum of the child before? Justify your answer. Assume angular momentum is taken about the axis of the merry-go-round.

+1 Angular Momentum is the same.

There are no external torque acting on child-merry-go-round system

OR
+1 IN Rotational Collisions Angular momentum is always conserved

- (c) Is the kinetic energy of the child-merry-go-round system after the child grabs onto the merry-go-round greater than, less than, or the same as the kinetic energy of the child before? Justify your answer.

KE decreases

The collision is perfectly inelastic (2 objects stick together) and KE is lost.

The angular momentum of the child before grabbing onto the merry-go-round is a constant value of mv_0R if taken about the axis of the merry-go-round. After the child grabs onto the merry-go-round, the rotational inertia of just the child can be modeled as having a value of mR^2 .

$L = mv_0R$
Child before

$I = mR^2$
Child after

(d) Briefly explain why the child's angular momentum before grabbing onto the merry-go-round is constant even though the child distance from the merry-go-round's axis is changing.

(e) Briefly explain why the child's rotational inertia after grabbing onto the merry-go-round can be modeled as mR^2 .

All of the child's mass can be treated as a distance R from the axis (center) of merry-go-round

(f) Derive an expression for the angular speed ω of the merry-go-round after the child grabs onto it in terms of $M, R, v_0, I,$ and m .

$I\omega = L$
 $\omega(I + mR^2) = mv_0R$

$\omega = \frac{mv_0R}{I + mR^2}$

$L = mv_0R$
Child's angular momentum
 $L = I\omega$

$\omega = \frac{L}{I = mR^2}$

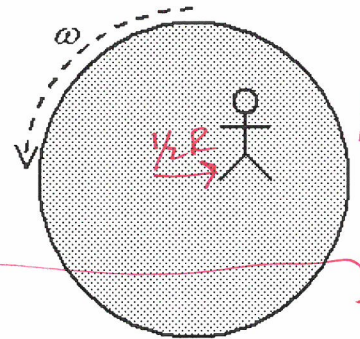
$L = \omega I$

Rotational $I = \frac{L}{\omega}$
 $L = \text{Angular momentum}$
 $\omega = \text{Angular velocity}$
 $\omega = \frac{L}{I}$

Child's angular momentum
 $L = mv_0R$

$\omega = \frac{L}{I}$

Now consider a different situation in which the merry-go-round platform is missing its handles. The child stands at a location that is a distance $\frac{1}{2}R$ from the center of the platform, and the platform is initially spinning with angular speed ω .



(g) State two specific actions that the child could do to increase the angular speed of the platform without the child touching anything other than the platform. For each actions you state, explain your reasoning.

move to the center, decreasing R

$\omega = \frac{L}{I}$

$\omega = \frac{mv_0R}{I}$ $\downarrow R$
 $\omega \uparrow$

$I = mR^2$

$\frac{mv_0R}{mR^2} = \frac{v_0}{R}$
 $\omega = \frac{v_0}{R}$

$L = I\omega$

$L = mv_0R$ Child Angular momentum

$I = I + mR^2$ Inertia of merry-go-round + child

$mv_0R = \omega(I + mR^2)$

$\omega = \frac{mv_0R}{I + mR^2}$

The angular momentum of the child before grabbing onto the merry-go-round is a constant value of mv_0R if taken about the axis of the merry-go-round. After the child grabs onto the merry-go-round, the rotational inertia of just the child can be modeled as having a value of mR^2 .

(d) Briefly explain why the child's rotational inertia after grabbing onto the merry-go-round can be modeled as mR^2 .

+1 All of the child's mass can be treated as a distance R from the axis (center) of the merry-go-round

(e) Derive an expression for the angular speed ω of the merry-go-round after the child grabs onto it in terms of $M, R, v_0, I,$ and m .

+1 $L = I\omega$

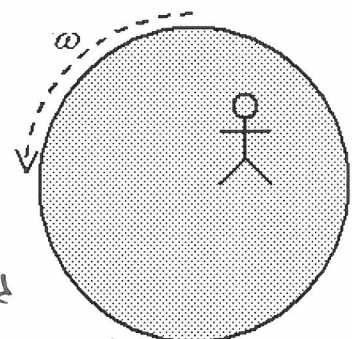
+1 $L = mv_0R$ Child Angular momentum

+1 $I = I + mR^2$ Interia of merry-go-round
 Interia of child

+1 $mv_0R = \omega(I + mR^2)$

+1 $\omega = \frac{mv_0R}{I + mR^2}$

Now consider a different situation in which the merry-go-round platform is missing its handles. The child stands at a location that is a distance $\frac{1}{2}R$ from the center of the platform, and the platform is initially spinning with angular speed ω .



(f) State two specific actions that the child could do to increase the angular speed of the platform without the child touching anything other than the platform. For each actions you state, explain your reasoning.

- +1 ① Child move to the center (axis) of merry-go-round
 - +1 This decreases the Rotation Inertia, which would require Angular Speed to increase to keep Angular Momentum constant
- ② The child could walk in a circle opposite the platform's rotation.
 - This will give the child less (negative) angular momentum requiring the platform to acquire more angular momentum to keep total angular momentum the same.