## AP Physics Unit 7 - Toque and Rotational Motion <br> Wkst - Rotational Motion-3 FRQ

## 1.

A double-pulley is built by welding together a solid disk of radius $r_{1}$ to a larger disk of radius $r_{2}$ so that the disks are coaxial. The double-pulley is supported by a horizontal axis that passes through the plane of the page. One string is wound several times around the smaller disk and a block of mass $m_{1}$ is hung from the free end of the string to the left of the disk. Another string is wound several times around the larger disk and a block of mass $m_{2}$ is hung from the free end of the string to the right of the disk.
(a) Suppose $m_{1}=9 \mathrm{~kg}$ and $r_{2}=3 r_{1}$. What mass must $m_{2}$ have in order for the system to remain at rest if the system were released from rest? Explain how you formulated your answer.


For parts (b) and (c), assume $m_{1}=m_{2}$. The system is released from rest.
(b) Which block rises upward once the system is released? Explain your reasoning.
(c) Which block has a greater magnitude of acceleration after the system is released, or do both blocks have the same magnitude of acceleration? Explain your reasoning.
(d) Does the gravitational potential energy of the $m_{1}-m_{2}$-pulley-Earth system increase, decrease, or remain constant as the blocks accelerate after the system is released? Explain your reasoning.
(e) Is the linear momentum of the $m_{1}-m_{2}$ system conserved during the interval after the system is released? Why or why not?

Block 2 and its string is removed. Block 1 is released with the system at rest at time $=0$ and allowed to fall a distance $d$ while the double-pulley rotationally accelerates. The time $t$ required for the block to fall the distance $d$ is recorded. The values of $m_{1}, r_{1}$, and $r_{2}$ are also measured using appropriate equipment.
(f) Explain how the measurements of $d, t, m_{1}, r_{1}$, and $r_{2}$ can be used to calculate the rotational inertia of the double-pulley. Your explanation will primarily consist of descriptive sentences but will necessarily require you to cite specific equations that model the situation and explain how the equations are used.
(g) How would the value of the rotational inertia of the pulley be different if Block 1 were removed and Block 2 were instead used in the experiment? Explain your reasoning.

## 2. Rotational Dynamics

A dense, heavy block of mass $m$ is connected to one end of a strong, thin, extremely light rod. The other end of the rod is fixed to a horizontal axis so that the center of the block is a distance $L$ from the center of the axis. The assembly is oriented so that the rod is vertical with the block on top (at point $A$ ) and released. Because the system was not perfectly balanced at the instant it was released, the rod-block system rotates clockwise so that the block passes through points $B, C$, and $D$ before reaching point $E$ where the rod is again vertical, this time with the block at the bottom. The system continues rotating through points $F, G$, and $H$ before coming momentarily to rest again at a point very close to and to the left of point $A$.
(a) On the dot below, draw and label the forces (not components) acting on the block when it passes through point $C$.

(b) Let $a$ be the tangential acceleration of the block, the rate at which the speed is changing with time. At which point(s) is the block located when the magnitude of $a$ is maximized? If there is a "tie" between two points where the magnitude of $a$ is maximized, state both points. In any case, justify your choice(s).
(c) At which point(s) is the block located when the magnitude of the block's linear speed maximized? If there is a "tie" between two points where speed is maximized, state both points. In any case, justify your choice(s).
(d) Let $\Delta t_{A C}$ represent the time it takes for the block to go from point $A$ to point $C, \Delta t_{B D}$ the time to go from point $B$ to point $D$, and $\Delta t_{C E}$ the time to go from point $C$ to point $E$. Rank these three time intervals and justify your ranking.

The rod is replaced by a new rod that is longer so that the length $L$ is doubled. The mass is released again from above the axis and rotates through the point directly below the axis. In this new situation, the time it takes for the block to move from above the axis to below the axis is $T_{2}$. In the previous situation, the time it took for the block to move from point $A$ to point $E$ was $T_{1}$. Angela believes that $T_{2}>T_{1}$, while Blake believes that $T_{2}<T_{1}$.
(e) Give a plausible reason why Angela could be correct. Be sure to cite how specific physical quantities are different in the new situation and discuss their relationships with other physical quantities.
(f) Give a plausible reason why Blake could be correct. Be sure to cite how specific physical quantities are different in the new situation and discuss their relationships with other physical quantities.
(g) At which location is the block, point $C$ or point $E$, when the magnitude of the force that the rod exerts on the block is maximized, or does the rod exert the same-magnitude force on the block at both points? Justify your answer.

## 3. Rotational Dynamics

A merry-go-round on a playground consists of a solid circular-disk platform supported by a frictionless axis at its center. The platform is fitted with handles on which children can grab and hang on as the platform rotates. The merry-go-round has mass $M$, radius $R$, and rotational inertia $I$. The merry-go-round is at rest when a child of mass $m$ runs with speed $v_{0}$ toward the merry-go-round, tangent to its edge. Upon reaching the edge, the child jumps on and hangs on to the merry-go-round, causing the child-merry-go-round system to have a constant angular speed $\omega$, as shown in the diagram.

(a) Is the linear speed of the child faster, slower, or the same after grabbing onto the merry-go-round when compared to the child's linear speed before? Justify your answer.
(b) Is the magnitude of the angular momentum of the child-merry-go-round system after the child grabs onto the merry-go-round greater than, less than, or the same as the angular momentum of the child before? Justify your answer. Assume angular momentum is taken about the axis of the merry-go-round.
(c) Is the kinetic energy of the child-merry-go-round system after the child grabs onto the merry-go-round greater than, less than, or the same as the kinetic energy of the child before? Justify your answer.

The angular momentum of the child before grabbing onto the merry-go-round is a constant value of $m v_{0} R$ if taken about the axis of the merry-go-round. After the child grabs onto the merry-go-round, the rotational inertia of just the child can be modeled as having a value of $m R^{2}$.
(d) Briefly explain why the child's rotational inertia after grabbing onto the merry-go-round can be modeled as $m R^{2}$.
(e) Derive an expression for the angular speed $\omega$ of the merry-go-round after the child grabs onto it in terms of $M, R, v_{0}, I$, and $m$.

Now consider a different situation in which the merry-go-round platform is missing its handles. The child stands at a location that is a distance $1 / 2 R$ from the center of the platform, and the platform is initially spinning with angular speed $\omega$.
(f) State two specific actions that the child could do to increase the angular speed of the platform without the child touching anything other than the platform.
For each actions you state, explain your reasoning.


