NATIONAL
MATH + SCIENCE
INITIATIVE

AP PHYSICS 1

Rotational Energy and Momentum

KEY

2016 EDITION
PRESENTER

Pre-Assessment Answers

(a) The plank would rotate counterclockwise, but it would not knock the guy out. This is because the bullet has less angular momentum than before, since it passes with the same speed but closer to the axis of rotation.

(b) If the bullet strikes point O, the plank would not rotate. The bullet only has angular momentum about point O if the bullet travels along a line that does NOT pass through the axis O.

(a) If the bullet strikes point C, the plank would rotate the other way. If Bond is very far away from the clothesline (so that the line from Bond to C is approximately parallel to the line from Bond to A), then the bullet has the same angular momentum as when the bullet hit A, and the clothesline would rotate around and slap the guy in the face, knocking him out.

(a) The plank acquires even more angular momentum than when the bullet stuck in point A. This is because the bullet goes from "counterclockwise" to "clockwise" angular momentum, a bigger change in angular momentum. This results in a bigger increase in angular momentum for the plank, so that the plank hits the guy even harder than originally.

Multiple-Choice Answers

M1. Answer: **D**

All three objects start with the same height and have the same mass, so all three objects have the same initial potential energy ($U_g = mgy$). That means all three objects end with the same kinetic energy because the kinetic energy comes from the potential energy.

M2. Answer: **A**

The solid sphere has the most mass distributed toward the middle, so the solid sphere has the least rotational inertia. All three objects have their potential energy divided between translational kinetic energy and rotational kinetic energy, but since the sphere has the least rotational inertia, the sphere "pays" the least energy into rotation and "keeps" more energy for translation, allowing it to travel faster at point Q.

M3. Answer: **A**

The only net force acting on the ball is the tension, and the tension points toward the center of the circle. Since tension points toward the center of the circle, the tension force exerts no torque. Since the tension force exerts no torque, the angular momentum must remain the same (torque is required to increase or decrease angular momentum).

Now angular momentum is $L = I\omega$, and pulling the ball inward would decrease its rotational inertia I, and therefore increase its angular speed ω as it travels around the circle. But kinetic energy is $K = \frac{1}{2}I\omega^2$. If I decreases and ω increases by the same factor, then kinetic energy increases because the square on ω does more to increase than the I does to decrease.

M4. Answer: **B**

If the ball is in the cart, the ball only has translational kinetic energy. If the ball rolls with the same speed, the ball has the same translational KE and also some rotational KE. This means the rolling ball has more total KE, and that greater amount of KE is what pushes the box a farther distance.

M5. Answer: **B**

The collision is not elastic, so some of the kinetic energy is transformed into other forms, making kinetic energy not conserved. However, there are no external forces on the system (the skater pushes the board, and the board pushes the skater, but those are internal to the system), so linear momentum p must be conserved. If there are no external forces, then there cannot be any external torques, so it is impossible to change the value of angular momentum L.

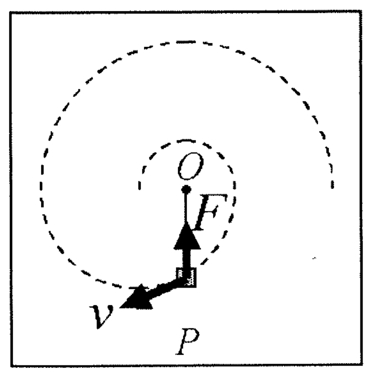
M6. Answer: **B&D**

There are three external forces acting on the disk-block system: the weight of the disk (acting downward at the center of the disk), the force of the axis supporting the disk (acting upward at the center of the disk), and the weight of the block (acting downward at the center of the block). Tension is internal to the disk-block system. To change angular momentum about point P, a force must exert torque about point P. To exert torque about P, a force must not point along a line on which P lies. The weight of the disk and the supporting force act along a line that does not pass through P, so those forces exert torque and change angular momentum. The weight of the block and the tension forces all act along a line that passes through P, and cannot exert torque or change angular momentum.

Free-Response Answers

F1. (7 points)

(a) 4 points



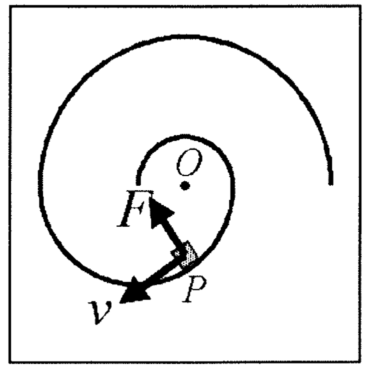
(1) The velocity vector is labeled v, starts on the block, and points in a direction down and to the left, tangent to the path in both diagrams for (a)-i and (b)-i.

(2) In the diagram for (a)-i, the force vector starts on the block, is labeled F, and points upward along the rope.

The kinetic energy decreases and the angular momentum stays the same.

(3) Kinetic energy decreases because there is an obtuse angle between F and v.
(4) Angular momentum remains the same because the force acts toward point O and does not cause a torque.

(b) 3 points



(5) The force labeled, starts on the block, and is perpendicular to the wall, directed up and left.

The kinetic energy remains the same and the angular momentum increases.

(6) The force is perpendicular to the velocity, so kinetic energy cannot increase or decrease.
(7) The force points in a direction that causes a clockwise torque about point O. This clockwise torque adds clockwise angular momentum to the already-clockwise-moving block.

F2. (12 points)

(a) 2 points

(1) The student makes any observation that indicates that the center of mass is where the bat would balance if placed on a fulcrum.
(2) The student clearly indicates that the center of mass can be found by balancing the bat on a fulcrum (or edge of a table) and that the location of the fulcrum is the center of mass.

(b) 3 points

(3) The student needs to measure the change in the height of the bat's center of mass as the bat swings down.
(4) The student indicates some way of measuring the speed of the end (or CM) of the bat at the bottom, such as using video analysis or photogates.
(5) The student indicates that the mass of the bat must be measured.

(c) 3 points

(6) The student explains how the instantaneous velocity of the end of the bat (or CM) can be measured based on the data they indicated for point (4).
(7) The student uses an equation such as $v = \omega r$ to get the angular speed of the bat at the bottom of its swing. The student needs to clearly indicate that r is the distance from the axis to the location on the bat where the instantaneous speed is measured.
(8) The student indicates that conservation of energy can be used to find the rotational inertia of the bat, including an equation such as $mgy = \frac{1}{2}I\omega^2$. The student needs to clearly indicate that y is the distance from the axis of rotation to the center of mass of the bat.

(d) 2 points

The student could answer yes or no, but their explanation must support their answer.

(9) The student connects their answer of "yes" or "no" to whether there is an external torque acting. (e.g. Yes angular momentum stays the same—no external torque acts, OR no angular momentum changes—an external torque acts).
(10) The student convincingly explains where the torque comes from, or why there is no torque.

Example Response: Yes it is the same. The only external force acting on ball-bat-batter system is the friction between the batter's feet and the dirt. This friction acts at the rotation axis, so the friction exerts no torque. If there is no torque, angular momentum cannot change.

Example Response: No, angular momentum changes. The ball experiences air resistance, which acts on the ball at a location far from the axis of rotation. The air resistance exerts a torque, and if a torque acts on the system, angular momentum changes.

(e) 2 points

(11) The student answers "no", and states that an external force acts on some component of the system.
(12) The student gives a valid example of an external force acting on the system. Examples include: friction on the batter, weight acting on the ball, or air resistance acting on the ball.

F3. (12 points)

(a) 3 points

(1) The angular momentum is torque times time, and the torque is rT, so the student obtains $L = rTt$ for part (a)-i.

(2) On part (a)-ii, the student combines $\tau = rT$ and $\alpha = \tau/I$ to obtain $\alpha = rT/I$. The student then uses $\omega = \alpha t$ along with the α found before to obtain $\omega = rTt/I$. (The student could also merely say $L = I\omega$ and $L = rTt$, so set these equal and solve to get $\omega = rTt/I$.) The student then plugs their expression for ω into the equation $K = \frac{1}{2}I\omega^2$ and ends with $K = \frac{r^2T^2t^2}{2I}$. (The student can also use the equation $K = \frac{L^2}{2I}$ and plug the answer for part (a)-i to get this same result.)

(3) The student uses the equation $v = \omega r$ and plugs in to obtain $v = \frac{Tr^2t}{I}$ on part (a)-iii.

(b) 5 points

(4) The student indicates that the rotational inertia does increase because rotational inertia increases as the mass of the rotating object moves away from the center.

(5) The student understands that more rotational inertia requires greater tension to rotate the apparatus.

(6) The student understands that tension is the force opposing mg and preventing the block from falling.

(7) The student recognizes that more opposing force results in less acceleration.

(8) The student uses an equation to support the idea that less acceleration results in less speed.

Example Response: When the weights move outward, the I increases and that makes it harder to turn. If the apparatus is harder to turn, then it takes more tension to rotate the apparatus.

The same tension pulls up on the block (opposite the block's acceleration). If the opposing force on the block increases, the block's acceleration decreases. Less acceleration means less speed according to $v = at$.

(c-i) 2 points

(9) The student recognizes that the system has greater rotational inertia between times 4 – 6 s.
(10) Greater rotational inertia results in a decreased change in motion.

(c-ii) 2 points

(11) The student's answer indicates an understanding that conservation of angular momentum applies to this situation.

(12) The student indicates that, as the rotational inertia increases, the rotational speed decreases if the angular momentum remains the same.

F4. (7 points)

(a) 2 points

In both cases the linear momentum AND angular momentum are conserved. This is because:

(1) There are no external forces acting on the system in either case, so linear momentum is conserved.

(2) If there are no external forces, then there are no external torques acting on the system either, so angular momentum is conserved.

(b) 5 points

In case 1, a greater amount of kinetic energy is transformed into other forms. Regardless of the student's answer, the student could get points for the following:

(3) Both systems have the same amount of momentum (OR their centers of mass move with the same speed) after the collision.

(4) If both systems have the same momentum (OR their CM's go the same speed), and they have the same mass, then they have the same translational kinetic energy.

(5) The system rotates in case 2 after the collision.

(6) This means that the apparatus has some rotational kinetic energy in case 2 after the collision.

(7) If both systems end with the same translational KE, but case 2 ends with some rotational KE, then more KE is lost during the collision in case 1.

Example Response: The way the puck hits in case 2 is going to cause the system to rotate after the collision, case 2 has some rotational kinetic energy after the collision. Both systems have the same translational kinetic energy after the collision because they both have the same total momentum and identical mass. That means case 2 has translational and rotational KE, but case 1 only has translational KE after the collision. That means case 1 must have lost more energy to other forms.