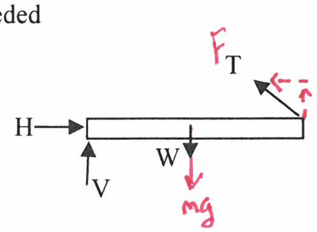
AP Physics – Unit 7 Toque and Rotational Motion
Wkst – PreExam – MC – Unit 7 - **KEY**

SECTION A – Torque and Statics

Solution | Answer

1. To balance the forces (Fnet=0) the answer must be A or D, to prevent rotation, obviously A would be needed — **A**

2. FBD — **B**



Since the rope is at an angle it has x and y components of force.
Therefore, H would have to exist to counteract T_x.
Based on $\tau_{net} = 0$ requirement, V also would have to exist to balance W if we were to chose a pivot point at the right end of the bar

3. Applying rotational equilibrium to each diagram gives — **D**

DIAGRAM 1: $(mg)(L_1) = (M_1g)(L_2)$
$L_1 = M_1(L_2)/m$
(sub this L_1) into the Diagram 2 eqn, and solve.

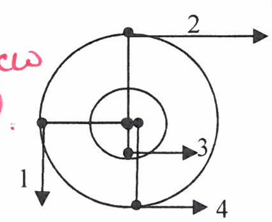
DIAGRAM 2: $(M_2g)(L_1) = mg(L_2)$
$M_2(L_1) = m(L_2)$

$M_2\left(\frac{M_1 L_2}{m}\right) = m L_2$
$M_2 M_1 = m^2$
$m = \sqrt{M_2 M_1}$

4. Find the torques of each using proper signs and add up. — **B**
+ (1) – (2) + (3) + (4)
+F(3R) – (2F)(3R) + F(2R) +F(3R) = 2FR

$\tau = F R_\perp$
$-2F(3R) + 2R(F) + 3RF + 3RF$
$-6FR + 2FR + 3FR + 3FR$
$= 2FR$

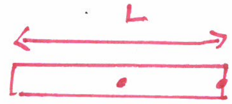
+ccw
-cw



5. Simply apply rotational equilibrium — **B**
$(m_1g) \cdot r_1 = (m_2g) \cdot r_2$
$m_1a = m_2b$

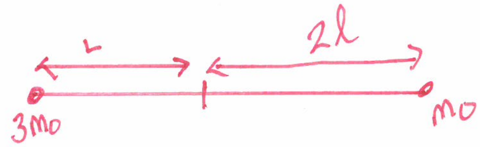
SECTION B – Rotational Kinematics and Dynamics

1. $I_{tot} = \Sigma I = I_0 + I_M = I_0 + M(\tfrac{1}{2}L)^2$ — **A**



$I_m = mR^2 \quad I_o = \frac{1}{2}Lm^2$

$I_{Total} = \Sigma I = I_o + I_m$
$= I_o + (\tfrac{1}{2}L)^2 m$
$= I_o + \tfrac{1}{4}L^2 m$



2. $\Sigma\tau = I\alpha$ where $\Sigma\tau = (3M_0)(l) - (M_0)(2l) = M_0 l$ and $I = (3M_0)(l)^2 + (M_0)(2l)^2 = 7M_0 l^2$ A

$\alpha = \frac{\Sigma\tau}{I}$ $\quad \tau = FR$ $\quad I = R^2 m$

$\Sigma\tau = -2L M_o + L3M_o$

$I = +(2L)^2 m_o + \frac{2}{3} M_o$

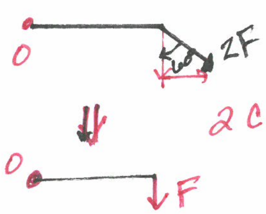
$\frac{M_o L}{4L^2 M_o + 3L^2 M_o}$

$\frac{M_o L}{7L^2 M_o}$

$\frac{1}{7L}$

3. $\tau_X = Fl$; $\tau_O = F_O L_O \sin\theta$, solve for the correct combination of F_O and L_O C

C



$2\cos 60° = 1$

4. Just as the tension in a rope is greatest at the bottom of as vertical circle, the force needed to maintain circular motion in any vertical circle is greatest at the bottom as the applied force must balance the weight of the object and additionally provide the necessary centripetal force C

5. $\Sigma F_{bottom} = F_{adhesion} - mg = F_{centripetal} = m\omega^2 r$ D

$F_{adh} - Mg = F_c$

$F_{adh} = F_c + mg$

$F_c = m\omega^2 r$

$= m\omega^2 r + mg$

Fc
mg
↑+

6. For one complete revolution $\theta = 2\pi$; $\omega^2 = \omega_0^2 + 2\alpha\theta$ C

$\theta = 2\pi$ $\quad$ 0 Rest

$\omega^2 = \cancel{\omega_o^2} + 2\alpha\phi$

$\omega^2 = 2\alpha(2\pi)$

$\omega^2 = 4\alpha\pi$

$\omega = 2\sqrt{\alpha\pi}$

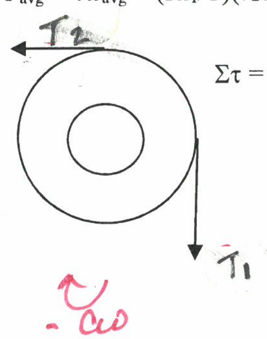
7. $\tau = \Delta L/\Delta t = (I\omega_f - 0)/T$ D

$\Delta L = \tau\Delta t$ $\quad L = I\omega$

$\tau = \frac{\Delta L}{\Delta t}$ $\quad \boxed{\tau = \frac{I\omega}{\Delta t}}$

SKIP 8. $P_{avg} = \tau\omega_{avg} = (I\omega_f/T)(\frac{1}{2}\omega_f)$ or $P_{avg} = \Delta K/T$ B

9. C



$\Sigma\tau = T_2R - T_1R = I\alpha$

$\Sigma\tau = FR$

$\Sigma\tau = T_2R - T_1R_1$

$\Sigma\tau = \alpha I$

$\alpha I = T_2R_1 - T_1R$

$\therefore a = 0$ $\quad F = m\cancel{a}^{0}$

10. If the cylinder is "suspended in mid air" (i.e. the linear acceleration is zero) then $\Sigma F = 0$ D

11. $\Sigma\tau = TR = I\alpha = ½\, MR^2\alpha$ which gives $\alpha = 2T/MR$ and since $\Sigma F = 0$ then $T = Mg$ so $\alpha = 2g/R$ the acceleration of the person's hand is equal to the linear acceleration of the string around the rim of the cylinder $a = \alpha R = 2g$ B

12. In order that the mass not slide down $f = \mu F_N \geq mg$ and $F_N = m\omega^2 R$ solving for μ gives $\mu \geq g/\omega^2 R$ A

F_f, F_c, F_N, mg

$F_s = \mu_s F_N$

$\Sigma F_y = F_f - mg = m\cancel{a}^{0}$

$F_f = mg$

$F_f = \mu_s F_n$

$F_f = \mu_s m\omega^2 r$

$\Sigma F_x = F_n - F_c = 0$

$F_n = F_c$

$F_n = m\omega^2 r$

$\mu_s \cancel{m}\omega^2 r = \cancel{m} g$

$\boxed{\mu_s = \dfrac{g}{\omega^2 r}}$

SECTION C – Rolling

1. $K_{tot} = K_{rot} + K_{trans} = ½\, I\omega^2 + ½\, Mv^2 = ½\, (2/5)MR^2\omega^2 + ½\, Mv^2 = (1/5)Mv^2 + ½\, Mv^2 = (7/10)Mv^2 = MgH$, solving gives $H = 7v^2/10g$ D

2. $Mgh = K_{tot} = K_{rot} + K_{trans}$, however without friction, there is no torque to cause the sphere to rotate so $K_{rot} = 0$ and $Mgh = ½\, Mv^2$ A

3. $Mgh = K_{tot} = K_{rot} + K_{trans} = ½\, I\omega^2 + ½\, Mv^2$; substituting v/r for ω gives $Mgh = ½(I/r^2 + M)v^2$ and solving for v gives $v^2 = 2Mgh/(I/r^2 + M)$, multiplying by r^2/r^2 gives desired answer D

4. The first movement of the point of contact of a rolling object is vertically upward as there is no side to side (sliding) motion for the point in contact A

SECTION D – Angular Momentum

1. $L_i = L_f$ so $I_i\omega_I = I_f\omega_f$ and since $I_f < I_i$ (mass more concentrated near axis), then $\omega_f > \omega_i$ — C
   The increase in ω is in the same proportion as the decrease in I, and the kinetic energy is proportional to $I\omega^2$ so the increase in ω results in an overall increase in the kinetic energy. Alternately, the skater does work to pull their arms in and this work increases the KE of the skater

2. $L = mvr_\perp$ where $r_\perp$ is the perpendicular line joining the origin and the line along which the particle is moving — B

3. $L = I\omega$ and since ω is uniform the ratio $L_{upper}/L_{lower} = I_{upper}/I_{lower} = 2mL^2/2(2m)(2L)^2 = 1/8$ — D

4. Since it is a perfectly inelastic (sticking) collision, KE is not conserved. As there are no external forces or torques, both linear and angular momentum are conserved — D

5. As there are no external forces or torques, both linear and angular momentum are conserved. As the type of collision is not specified, we cannot say kinetic or mechanical energy *must* be the same. — A/B