AP Physics – Unit 7

Pre-Exam – FRQ – Unit 7 -KEY

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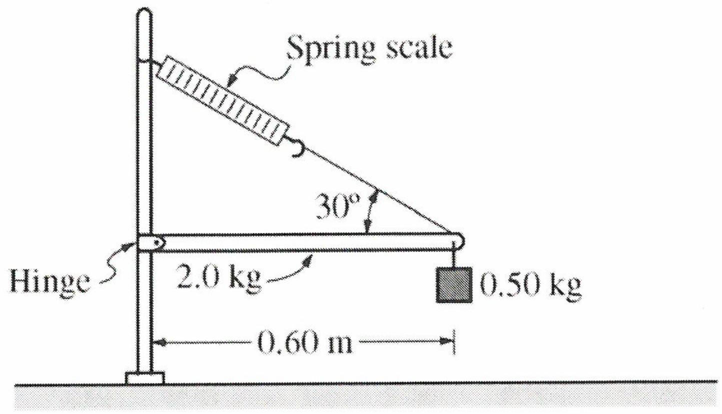
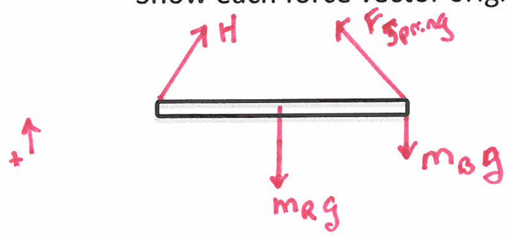


1. The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

   a. On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.



$m_B = 0.50\,kg$ $\quad \theta = 30.°$

$m_R = 2.0\,kg$

$R = 0.60\,m$

   b. Calculate the reading on the spring scale.'

F_r, F_{ry}, F_{rx}

$\Sigma \tau_{net} = \emptyset$

$\Sigma \tau_{net} = \frac{1}{2} R M_R g + R m_B g - F_{spring} \sin\theta\, R = \emptyset$

$F_{spring} = \dfrac{\frac{R}{2} M_R g + R M_B g}{R \sin\theta} = \dfrac{g \cancel{R} (\frac{m_R}{2} + m_B)}{\cancel{R} \sin\theta} = \dfrac{(9.8\,m/s^2)\left[\left(\frac{2.0\,kg}{2}\right) + 0.50\,kg\right]}{\sin 30°}$

$\boxed{F_{Spring} = 29\,N}$

The rotational inertia of a rod about its center is $\frac{1}{12}ML^2$, where *M* is the mass of the rod and *L* is its length

   c. Calculate the rotational inertia of the rod-block system about the hinge.

$I_s = I_r + I_B$

$I_r = I_{r,cm} + mR^2$ (Parrell Axis Theorem)

$I_{r,cm} = \frac{1}{12} M L^2$

$R = R/2$ center

$I_r = \frac{1}{12} m R^2 + m\left(\frac{R}{2}\right)^2 = \frac{1}{12} mR^2 + \frac{1}{4} mR^2 = \frac{4}{12} mR^2$ $\quad 3/12$

$I_B = mR^2$

$I_s = I_r + I_B$

$= \frac{4}{12} m_R R^2 + m_B R^2$

$= \frac{1}{3}(2.0\,kg)(0.60\,m)^2 + (0.50\,kg)(0.60\,m)^2$

$\boxed{I_s = 0.42\,kg\,m^2}$

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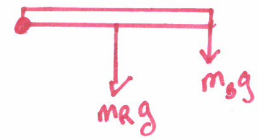
d. If the cord that supports the rod is cut near the end of the rod, calculate the initial angular acceleration of the rod-block system about the hinge. $\alpha = ?$



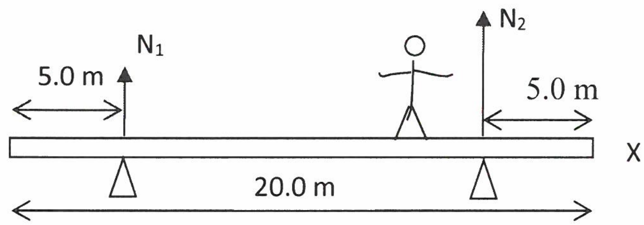
$\Sigma \tau_{net} = I\alpha$

$m_R g \frac{R}{2} + m_B g R = I\alpha$

$\alpha = \dfrac{gr\left(\frac{m_{Rod}}{2} + m_B\right)}{I} = \dfrac{(9.8\,m/s^2)(0.60\,m)\left[\frac{2.0\,kg}{2} + 0.50\,kg\right]}{0.42\,kg \cdot m^2}$

$\boxed{\alpha = 21\ \text{Radians}/s^2}$

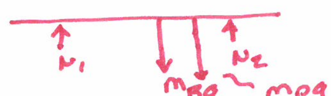
2. The diagram below shows a beam of length 20.0 m and mass 40.0 kg resting on two supports placed at 5.0 m from each end.



$M_{Beam} = 40.0\,kg$
$M_{person} = 50.0\,kg$

A person of mass 50.0 kg stands on the beam between the supports. The reaction forces at the supports are shown.

a. State the value of $N_1 + N_2$



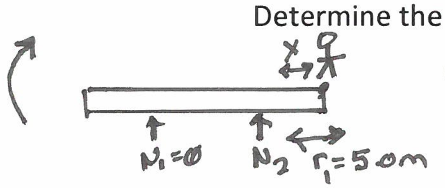
$\Sigma F_y = m\cancel{a}^{0}$

$\Sigma F_y = N_1 + N_2 - m_B g - m_P g = 0$

$N_1 + N_2 = g(m_B + m_P) = 9.8\,m/s^2\,(40.0\,kg + 50.0\,kg)$

$\boxed{N_1 + N_2 = 882\,N}$

b. The person now moves toward the X end of the beam to the position where the beam just begins to tip and reaction force N_1 becomes zero as the beam starts to leave the left support. Determine the distance of the girl from the end X when the beam is about to tip.



How far from End will it start to tip? (x)

Rotational Equilibrium

$M_{Beam}\, r_1 = M_{person}\, X$

$X = \dfrac{M_{Beam}\, r_1}{m_{person}} = \dfrac{(40.0\,kg)(5.0\,m)}{50.0\,kg}$

$X = 4.0\,m$

This is the distance from N_2

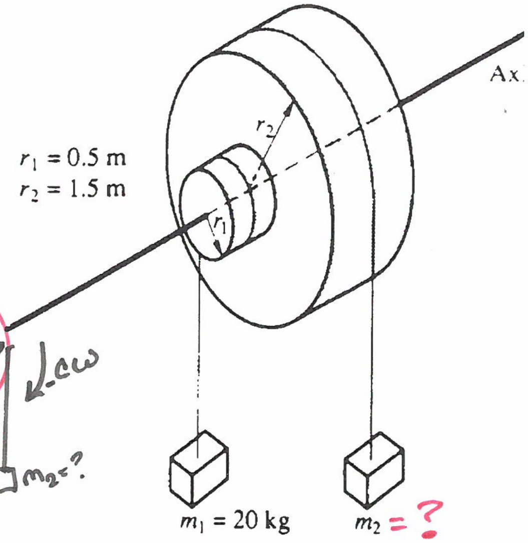
Distance from <u>End</u>

$5.0\,m - 4.0\,m$

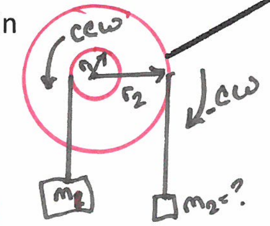
$\boxed{1.0\,m \text{ girl from End}}$

Key

3. Two masses. m_1 and m_2, are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively, as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is $I = 45\ kg \bullet m^2$. Also $r_1 = 0.5$ meter, $r_2 = 1.5$ meters, and $m_1 = 20$ kilograms.



a. Determine m_2 such that the system will remain in equilibrium



$\Sigma \tau_{net} = \emptyset$ Equilibrium - Not spinning

$\tau_1 - \tau_2 = 0$

$\tau_1 = \tau_2$

$m_1 r_1 g = \overset{?}{m_2} r_2 g$

$m_2 = \dfrac{m_1 r_1}{r_2}$

$m_2 = \dfrac{(20kg)(0.5m)}{1.5m}$

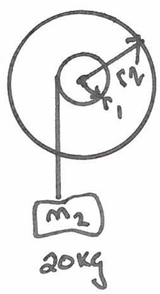
$m_2 = 6.7\ kg$

$\boxed{m_2 = 7\ kg}$

The String is Rolling off Pulley at same Rate m_2 is falling (a) [αr]

The mass m_2 is removed and the system is released from rest.

b. Determine the angular acceleration of the cylinders



$I = 45\ kg\,m^2$

$\alpha = ?$

$\tau = I\alpha$

$\tau = F_T R_1$

$F_T R_1 = I\alpha$

$\overset{?}{\alpha} = \dfrac{\overset{?}{F_T} R_1}{I}$ Need F_T

F_T ↑ mg ↓ ↓+

$\Sigma F_y = mg - F_T = ma$

$a = \alpha r_1$

$F_T = mg - m_2 \alpha r_1$

2 unknowns Big Algebra!!

$\alpha = \dfrac{(m_2 g - m_2 \alpha r_1) R_1}{I}$

$\alpha = \dfrac{m_2 g R_1 - m_2 \alpha R_1^2}{I}$

$m_2 \alpha R_1^2 + \alpha I = m_2 g R_1$

$\alpha(m_2 R_1^2 + I) = m_2 g R_1$

$\alpha = \dfrac{m_2 g R_1}{(m_2 R_1^2 + I)}$

$= \dfrac{(20kg)(9.8 m/s^2)(0.5m)}{(20kg(0.5m)^2 + 45\ kg\,m^2)}$

$\boxed{\alpha = 1.96\ Rad/s}$

c. Determine the tension in the cable supporting m_1

$F_T = ?$

$F_T R_1 = I\alpha$ (from above)

$F_T = \dfrac{I\alpha}{R_1} = \dfrac{(45\ kg \cdot m^2)(1.96\ Rad/s)}{(0.50m)}$

$= 176.4$

$\boxed{F_T = 200\ N}$

d. Determine the linear speed of m_1 at the time it has descended 1.0 meter.

$V = ?$

$\Delta y = 1.0$ ↑+

Kinematics eqns or could also do $PE + KE = PE_f + KE_f$

$V_y^2 = \cancel{V_{y0}}^{\,\to 0\ Rest} + 2a\Delta y$

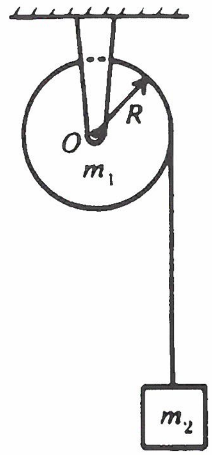
$a = \alpha r$

$V_y = \sqrt{2\alpha r \Delta y}$

$V_y = \sqrt{2(1.96)(0.5)(1.0m)}$

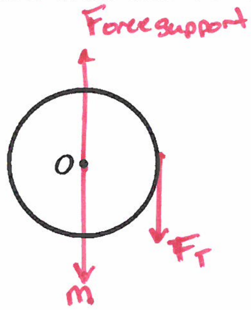
$\boxed{V_y = 1.4\ m/s}$

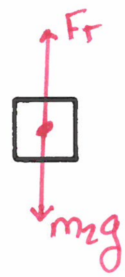
Key



4. A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis through O. The moment of inertia of the cylinder about the axis is $I = ½m_1R^2$. A block of mass m_2, suspended by a cord wrapped around the cylinder as shown above, is released at time t = 0.

   a. On the diagram below draw and identify all of the forces acting on the cylinder and on the block.





$\downarrow +$

   b. In terms of m_1, m_2, R, and g, determine each of the following.
      i. The acceleration of the block
      ii. The tension in the cord

Cylinder

ii) $\Sigma \tau = I\alpha$

$I = \frac{1}{2} m r^2$ (Rod)

$\alpha = \frac{a}{r}$

$F_T \cancel{r} = \frac{1}{2} m \cancel{r}^2 \left(\frac{a}{\cancel{r}}\right)$

$\boxed{F_T = \frac{1}{2} m_1 a}$

Block

i) $\Sigma F = ma$

$m_2 g - F_T = m_2 a$

$F_T = \frac{1}{2} m a$

$m_2 g - \frac{1}{2} m_1 a = m_2 a$

$m_2 g = a\left(\frac{1}{2} m_1 + m_2\right)$

$\boxed{a = \frac{m_2 g}{\left(\frac{1}{2} m_1 + m_2\right)}}$

Key

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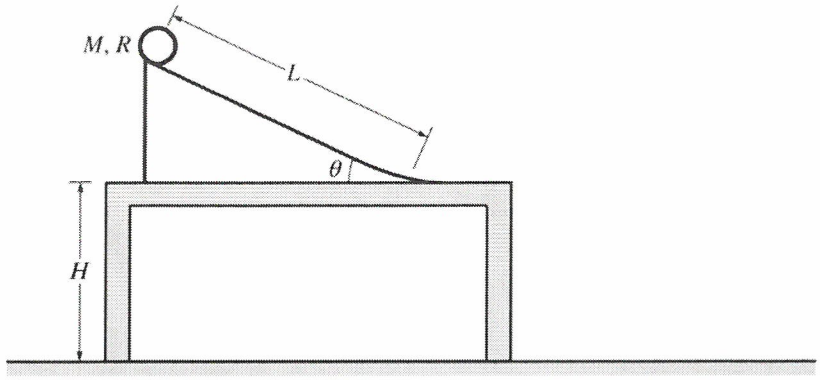
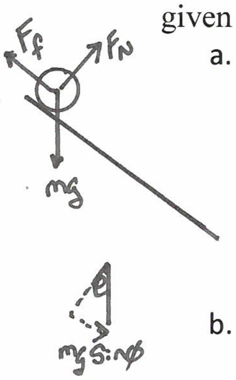


5. A thin hoop of mass M, radius R, and rotational inertia MR^2 is released from rest from the top of the ramp of length L above. The ramp makes an angle θ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height H above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.



a. Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.

Given:
$a = ?$
$I = MR^2$
$\alpha = \frac{a}{R}$

$\Sigma \tau = I\alpha$
$\cancel{R} F_f = M\cancel{R}^{\cancel{2}} \left(\frac{a}{\cancel{R}}\right)$
$F_f = ma$

$\Sigma F = ma$
$mg\sin\theta - F_f = ma$
$\cancel{m}g\sin\theta - \cancel{m}a = \cancel{m}a$
$2a = g\sin\theta$

$\boxed{a = \frac{g}{2}\sin\theta}$

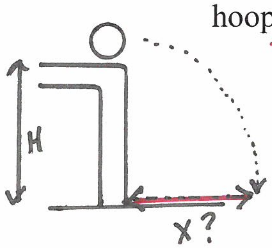
b. Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.

Given: $V = ?$
$a = \frac{1}{2}g\sin\theta$

Soln:
$V_f^2 = \cancel{V_0^2}^{\,0 \text{ Rest}} + 2a\Delta x$
$\Delta x = L$
$a = \frac{1}{2}g\sin\theta$
$V_f^2 = \cancel{2}\left(\frac{1}{\cancel{2}}g\sin\theta\right)L$

$\boxed{V_f = \sqrt{g\sin\theta\, L}}$

c. Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.



This problem always
How long in Air?
Then how far!

① $y = \cancel{y_0}^{\,0} + \cancel{V_{y0}t}^{\,0} + \frac{1}{2}gt^2$
$H = \frac{1}{2}gt^2$
$t = \sqrt{\frac{2H}{g}}$

② $x = \cancel{x_0}^{\,0} + V_x t + \cancel{\frac{1}{2}at^2}^{\,0}$
$V_x = V_f = \sqrt{g\sin\theta L}$
$t = \sqrt{\frac{2H}{g}}$
$x = \sqrt{\frac{2H}{g}}\sqrt{g\sin\theta L}$
$\boxed{x = 2H\sin\theta L}$

d. Suppose that the hoop is now replaced by a disk having the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part c. for the hoop?

Less than____ The same as____ Greater than__X__

Briefly justify your response

The disk has a smaller moment of Inertia, so it has a larger acceleration & will reach a higher speed at the Bottom of the ramp. ∴ Causing it to go farther

Key

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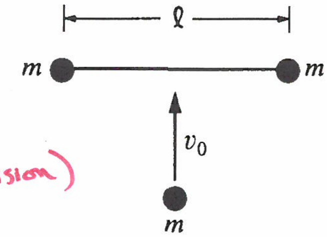
6. A space shuttle astronaut in a circular orbit around the Earth has an assembly consisting of two small dense spheres, each of mass m, whose centers are connected by a rigid rod of length l and negligible mass. The astronaut also has a device that will launch a small lump of clay of mass m at speed v_0. Express your answers in terms of m, v_0 l. and fundamental constants.

a

i) $K_f = ?$

$K_f = \frac{1}{2} m V_f^2$

But what is V_f of ? (After collision) 3M



a. Initially, the assembly is "floating" freely at rest relative to the cabin, and the astronaut launches the clay lump so that it perpendicularly strikes and sticks to the midpoint of the rod, as shown above.

i. Determine the total kinetic energy of the system (assembly and clay lump) after the collision. Need V_f use momentum Collision 1st

① $P_i = P_f$

$m_i V_i = m_f V_f$

Rest; No V_i + $\uparrow V_0$ =

$m_i(0) + m V_0 = 3mV_f$

$\cancel{m} V_0 = 3\cancel{m} V_f$

$V_f = \frac{1}{3} V_0$

② $K_f = \frac{1}{2} m V_f^2$

$= \frac{1}{2} 3m \left(\frac{1}{3} V_0\right)^2$

$= \frac{3}{2} m \frac{1}{9} V_0^2$

$\boxed{K_f = \frac{1}{6} m V_0^2}$

ii. Determine the change in kinetic energy as a result of the collision.

$\Delta KE = KE_f - KE_i$

$KE_f = \frac{1}{6} m V_0^2$

$KE_i =$ Rest + $\uparrow V_0$

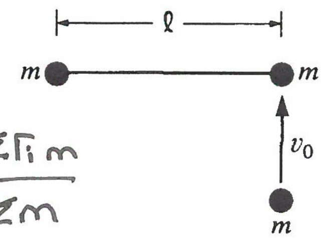
$= \frac{1}{2} m V_0^2$

$\Delta KE = \frac{1}{6} m V_0^2 - \frac{1}{2} m V_0^2$

$\left(\frac{3}{6}\right)$

$\boxed{\Delta KE = -\frac{1}{3} m V_0^2}$

b. The assembly is brought to rest, the clay lump removed, and the experiment is repeated as shown above, with the clay lump striking perpendicular to the rod but this time sticking to one of the spheres of the assembly. $V_0 = \emptyset$ Rest



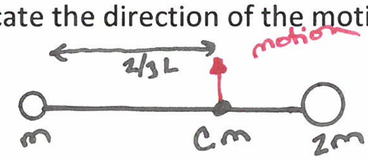
i. Determine the distance from the left end of the rod to the center of mass of the system (assembly and clay lump) immediately after the collision. (Assume that the radii of the spheres and clay lump are much smaller than the separation of the spheres.) Center mass?

$r_{cm} = \frac{\Sigma r_i m}{\Sigma m}$

$= \frac{m(0) + 2mL}{3m}$

$\boxed{r_{cm} = \frac{2}{3} L}$

ii. On the figure above, indicate the direction of the motion of the center of mass immediately after the collision.



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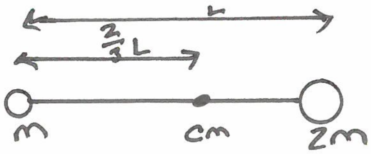


iii. Determine the speed of the center of mass immediately after the collision.

$V_{cm} = ?$

$P_i = P_f$

P_i

$2m(0) + mV_0 = 3mV_f$

$\cancel{m}V_0 = 3\cancel{m}V_f$

$V_f = \frac{1}{3}V_0$

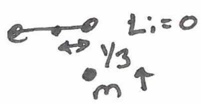
iv. Determine the angular speed of the system (assembly and clay lump) immediately after the collision.

$\omega = ?$

$L_i = L_f$ Angular momentum Conserved

$L_i = I\omega = mVR_0$

$L_f = I\overset{?}{\omega}$



$L_i = mV_0\frac{1}{3}L$

$L_i = L\omega$

$\frac{1}{3}LmV_0 = I\omega$ — from cm

$I = mR^2$

$= 2m\left(\frac{1}{3}L\right)^2 + m\left(\frac{2}{3}L\right)^2$

$= 2m\frac{1}{9}L^2 + m\frac{4}{9}L^2$

$= \frac{2}{9}mL^2 + \frac{4}{9}mL^2$

$\boxed{I = \frac{6}{9}mL^2 = \frac{2}{3}mL^2}$

$\frac{1}{3}\cancel{L}\cancel{m}V_0 = \frac{2}{3}\cancel{m}L^{\cancel{2}}\omega$

$V_0 = 2L\omega$

$\boxed{\omega = \frac{V_0}{2L}}$

v. Determine the change in kinetic energy as a result of the collision.

$\Delta KE = ?$

$KE_i = \frac{1}{2}mV_0^2$

$KE_f = \text{Tran} + \text{Rot}$

$= \frac{1}{6}mV_0^2 + \frac{1}{2}I\omega^2$

(a i)

$I = \frac{2}{3}mL^2$

$\omega = \frac{V_0}{2L}$

$\Delta KE = KE_f - KE_i$

$= \frac{1}{6}mV_0^2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}m\cancel{L^2}\right)\left(\frac{V_0}{\cancel{2L}}\right)^2 - \frac{1}{2}mV_0^2$

$= \frac{1}{6}mV_0^2 + \frac{1}{12}mV_0^2 - \frac{1}{2}mV_0^2$

$= \frac{2}{12}mV_0^2 + \frac{1}{12}mV_0^2 - \frac{6}{12}mV_0^2$

$= -\frac{3}{12}mV_0^2$

$\boxed{\Delta KE = -\frac{1}{4}mV_0^2}$