AP Physics - Unit 3 - Circular Motion

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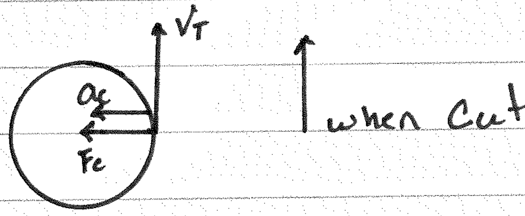
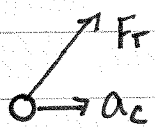
WKst: Circular Motion Packet

①



②



③ Given:

$m = 2400\,kg$

$r = 35m$

$t = 23.2s$

$1\,rev = 2\pi$

Ⓐ $V_T = ?$

$$V_T = \frac{r\Delta\theta}{\Delta t}$$

$\theta = 2\pi$

$$V_T = \frac{2\pi r}{\Delta t} = \frac{2\pi(35m)}{23.2sec}$$

$$\boxed{V_T = 9.5\,m/s}$$

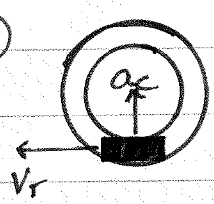
Ⓑ $a_c = ?$

$$a_c = \frac{V_T^2}{r}$$

$$= \frac{(9.5m/s)^2}{35m}$$

$$\boxed{a_c = 2.6\,m/s^2}$$

Ⓓ



Ⓒ $F = ma_c$ $F = ?$

$$= (2400kg)(2.6m/s^2)$$

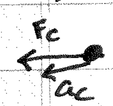
$$\boxed{F = 6240\,N}$$

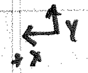
④ Given:

$V_T = 1.0\,m/s$

$R = 1.08\,m$

$M = 40.0\,kg$





Ⓐ $a_c = ?$

$$a_c = \frac{V_T^2}{R} = \frac{(1.0m/s)^2}{1.08m}$$

$$\boxed{a_c = 0.93\,m/s^2}$$

Ⓑ $F = ?$ $\Sigma F_x = F_c = ma_c$

$$F = (40.0kg)(0.93m/s^2)$$

$$\boxed{F = 37N}$$

Ⓒ Freq = ? $V_T = \frac{r\Delta\theta}{\Delta t}$

$\Delta t = ?$ $\Delta\theta = 2\pi$

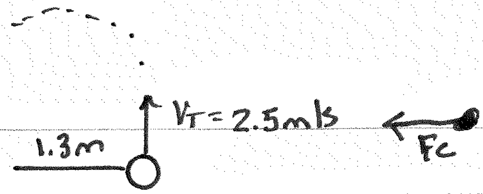
$$V_T = \frac{2\pi r}{t}$$

$$t = \frac{2\pi r}{V_T}$$

$$t = \frac{2\pi(1.08m)}{1.0m/s}$$

$$t = 6.79s$$

$$f = 1/t = \frac{1}{6.79} = \boxed{0.14\,Hz}$$



⑤ Given:

$m = 1.8\ kg$

$r = 1.3m$

$V_T = 2.5m/s$

A) $t = ?$

B) $f = ?$

C) $a_c = ?$

D) $F_T = ?$

A) $V_T = \frac{r\Delta\theta}{t}$ $\quad \theta = 2\pi$

$V_T = \frac{2\pi r}{t}$

$t = \frac{2\pi r}{V_T}$

$= \frac{2\pi(1.3m)}{2.5m/s}$

$\boxed{t = 3.27s}$

B) $f = \frac{1}{T}$

$f = \frac{1}{3.27s}$

$\boxed{f = 0.31\ Hz}$

C) $a_c = \frac{V_T^2}{r}$

$= \frac{(2.5m/s)^2}{1.3m}$

$\boxed{a_c = 4.8m/s^2}$

D) $F_T = F_c$

$= ma_c$

$= (1.8kg)(4.8m/s^2)$

$\boxed{F_T = 8.7N}$

⑥ A) $m \times 3$

$F_c = ma_c$

$a_c = \frac{V_T^2}{r}$

$F_c = \frac{mV_T^2}{r}$ $\quad 3 \times m$

$F_c = \frac{3m\ V_T^2}{r}$ $\quad V_T = 1$, $r = 1$, $m = 1$

$\therefore 3F_c \Rightarrow F_c = 3x$

B) $r \times 2$

$F_c = \frac{mV_T^2}{r}$

$F_c = \frac{(1)(1)^2}{2}$

$F_c = \frac{1}{2}x$

C) $2 \times V$

$F_c = \frac{mV_T^2}{r}$

$= \frac{1(2)^2}{1}$

$F_c = 4x$

D) $m \times \frac{1}{2}$, $r \times 4$

$F_c = \frac{mV_T^2}{r}$

$= \frac{(\frac{1}{2})(1)^2}{4}$

$F_c = \frac{1}{8}$

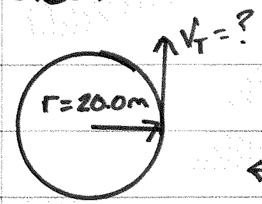
E) Period $\times 2 \Rightarrow f = \frac{1}{T}$

$t = \frac{1}{2}$

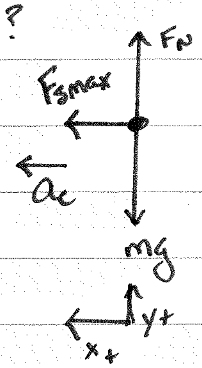
$F_c = \frac{m(\frac{1}{2})^2}{r} = \frac{(1)(\frac{1}{2})^2}{1}$

$F_c = \frac{1}{4}$

7 Given



$\mu_s = 0.600$



$\Sigma F_x = F_{smax} = ma_c$

$\Sigma F_y = F_N - mg = m\cancel{a}^{0}$

$F_n = mg$

$F_{smax} = \mu_s F_n$

$= \mu_s mg$

$a_c = \frac{V_T^2}{r}$

$\mu_s \cancel{m} g = \cancel{m} \frac{V_T^2}{r}$

$\mu_s g = \frac{V_T^2}{r}$?

$V_T^2 = \mu_s g r$

$= \sqrt{(0.600)(9.8 m/s^2)(20.0m)}$

$\boxed{V_T = 10.8 m/s}$

Or

$F_{net} = F_c$

$F_c = m\frac{V_T^2}{R}$

$F_{net} = F_{smax}$

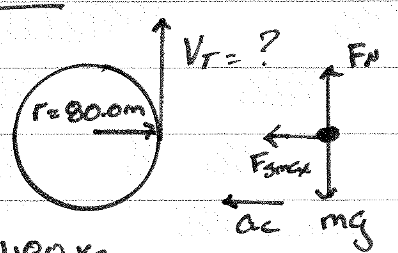
$F_{smax} = m\frac{V_T^2}{r}$

$F_{smax} = \mu_s mg$

$\mu_s \cancel{m} g = \cancel{m} \frac{V_T^2}{r}$

$V_T^2 = \mu_s g r$

(8) Given:



m= 1180 kg

$U_s = 0.80$

$\Sigma F_x = F_{smax} = ma_c$ $\qquad$ $\Sigma F_y = F_N - mg = m\cancel{a}$

$F_{smax} = U_s F_N$ $\qquad$ $F_N = mg$

$= U_s mg$

$a_c = \frac{V_T^2}{r}$

$U_s \cancel{m}g = \cancel{m}\frac{V_T^2}{r}$

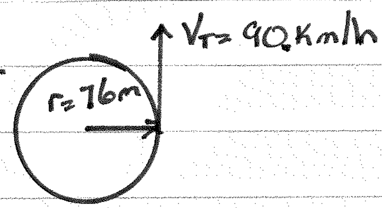
$V_T = \sqrt{U_s R g}$

$= \sqrt{(0.80)(80.0m)(9.8 m/s^2)}$

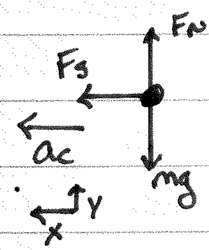
$\boxed{V_T = 25 m/s}$

A) yes, the mass of the car has no effect since m cancels out

(9) Given:



$U_s = ?$



$\left(\frac{90\ km}{hr}\right)\left(\frac{1000m}{1km}\right)\left(\frac{1hr}{3600s}\right) = 25 m/s$

$\Sigma F_x = F_{smax} = ma_c$ $\qquad$ $\Sigma F_y = F_n - mg = m\cancel{a}$

$F_{smax} = \overset{?}{U_s} F_n$ $\qquad$ $F_n = mg$

$= U_s mg$

$a_c = \frac{U_T^2}{r}$

$U_s \cancel{m}g = \cancel{m}\frac{V_T^2}{r}$

$U_s = \frac{V_T^2}{gr}$

$U_s = \frac{(25 m/s)^2}{(9.8 m/s^2)(76m)}$

$U_s = 0.84$

or $\quad F_{net} = F_c$

$F_{net} = F_s = U_s mg$

$F_c = m\frac{V_T^2}{r}$

$U_s \cancel{m}g = \cancel{m}\frac{V_T^2}{r}$

$U_s = \frac{V_T^2}{rg}$

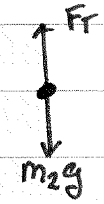
(10) Given:

$f = ?$

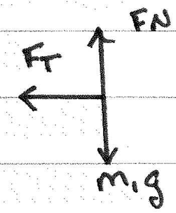
m_1, m_2, r

Soln:

Hanging



Table



$$\Sigma F_y = F_T - m_2 g = \emptyset$$

$$F_T = m_2 g$$

$$F_{net} = F_c$$

$$F_{net} = F_T$$

$$F_c = m_1 \frac{v_T^2}{r}$$

$$F_T = m_1 \frac{v_T^2}{r}$$

want f

$$v_T = 2\pi r f$$

$$F_T = m_1 \frac{4\pi^2 r^{\cancel{2}} f^2}{\cancel{r}}$$

$$F_T = F_T$$

$$m_2 g = m_1 4\pi^2 r f^2$$

$$f^2 = \frac{m_2 g}{m_1 4\pi^2 r}$$

$$f = \sqrt{\frac{m_2 g}{m_1 4\pi^2 r}}$$

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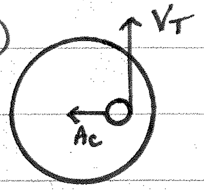
11 A



Given:

$m = 0.0050\ kg$

$r = 0.14\ m$

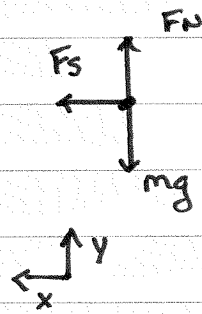
$T = 1.5\ s$

B $V_T = ?$

$$V_T = \frac{2\pi r}{T} = \frac{2(\pi)(0.14m)}{1.5s}$$

$$\boxed{V_T = 0.59\ m/s}$$

C coin



$\mu_s = 0.50$

$V_T = ?$

$$F_{net} = F_c$$

$$F_c = \frac{mV_T^2}{r}$$

$$F_{net} = F_s = \mu_s mg$$

$$\mu_s \cancel{m} g = \cancel{m}\frac{V_T^2}{r}$$

$$V_T^2 = \mu_s g r$$

$$= \sqrt{(0.50)(9.8 m/s^2)(0.14m)}$$

$$\boxed{V_T = 0.83 m/s}$$

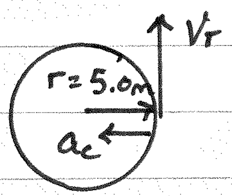
D The mass cancels, so changing the mass would have no effect

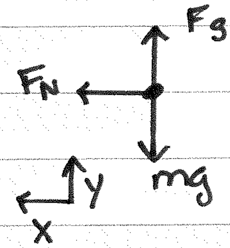
13 Given:

$m = 80.0\,kg$

$\mu_s = 0.40$

$f = ?$





$F_{net} = F_c$

$F_{net} = F_n$

$F_c = m\frac{V_T^2}{r}$

$F_n = m\frac{V_T^2}{r}$

$\Sigma F_y = F_s - mg = m\cancel{a}^{0}$

$F_s = mg$

$F_s = \mu_s F_n$

$\mu_s F_N = mg$

$F_N = \frac{mg}{\mu_s}$

$$\frac{\cancel{m}g}{\mu_s} = \frac{\cancel{m}V_T^2}{r}$$

$$V_T^2 = \frac{gr}{\mu_s}$$

$V_T = 2\pi r f$ want f

$$4\pi^2 r^2 f^2 = \frac{gr}{\mu_s}$$

$$f^2 = \frac{g\cancel{r}}{4\pi^2 r^{\cancel{2}} \mu_s}$$

$$f = \sqrt{\frac{9.8\,m/s^2}{4\pi^2(5.0\,m)(0.40)}}$$

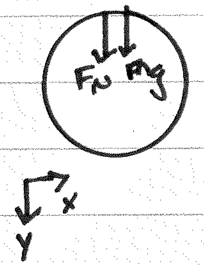
$$\boxed{f = 0.35\,Hz}$$

Vertical Loop

(14) Given:

$V_T = ?$

$r = 8.4\,m$



$F_{net} = F_c$

$F_{net} = \cancel{F_N} + mg$ $\xrightarrow{} 0$

$F_n = 0$ minimum speed

$F_c = m\frac{V_T^2}{r}$

$\cancel{m}g = \cancel{m}\frac{V_T^2}{r}$

$V_T^2 = rg$

$V_T = \sqrt{(8.4m)(9.8\,m/s^2)}$

$\boxed{V_T = 9.1\,m/s}$

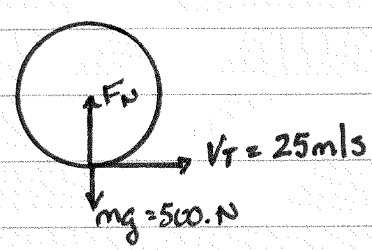
(15.)

Given:

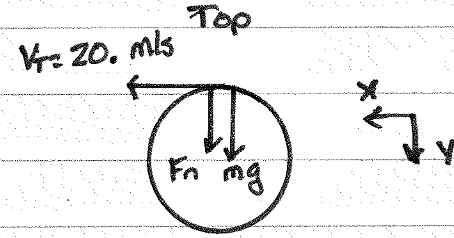
$r = 30.0\,m$

Weight = 500. N

Bottom



Top



$F_{net} = F_c$

$F_c = m\frac{V_T^2}{r}$

$F_{net} = F_N - mg$

$F_N - mg = m\frac{V_T^2}{r}$

$F_N = mg + m\frac{V_T^2}{r}$

$F_g = mg$

$m = \frac{500\,N}{9.8} = 51\,kg$

$F_N = 500.\,N + (51\,kg)\left(\frac{(25\,m/s)^2}{30.0\,m}\right)$

$= 500.\,N + 1060\,N$

$\boxed{F_N = 1560\,N}$

$F_{net} = F_c$

$F_c = m\frac{V_T^2}{r}$

$F_{net} = F_N + mg$

$F_N + mg = m\frac{V_T^2}{r}$

$F_N = \frac{mV_T^2}{r} - mg$

$= \frac{(51\,kg)(20.\,m/s)^2}{30.0\,m} - 500.\,N$

$= 680\,N - 500.\,N$

$\boxed{F_N = 180\,N}$

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$r = 25m$

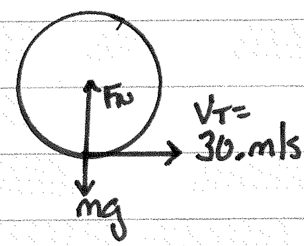
weight $= 600N$

$F_g = mg$

$m = \frac{600N}{9.8 m/s^2}$

$m = 61.2 kg$

Bottom



$F_{net} = F_c$

$F_{net} = F_N - mg$

$F_c = m\frac{V_T^2}{r}$

$F_N - mg = m\frac{V_T^2}{r}$

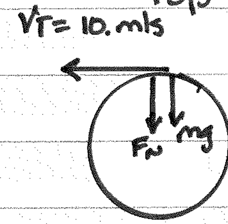
$F_N = mg + m\frac{V_T^2}{r}$

$= 600.N + \frac{(61.2kg)(30.m/s)^2}{25m}$

$= 600.N + 2200N$

$\boxed{F_N = 2800N}$

Top



$F_{net} = F_c$

$F_c = m\frac{V_T^2}{r}$

$F_{net} = mg + F_N$

$mg + F_N = m\frac{V_T^2}{r}$

$F_N = m\frac{V_T^2}{r} - mg$

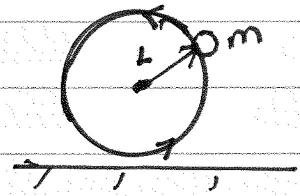
$= (61.2kg)\left(\frac{(10.m/s)^2}{25m}\right) - 600.N$

$= 240N - 600.N$

$\boxed{F_N = -366N}$

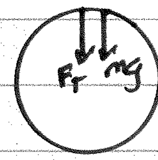
Not fast enough won't make it around the Loop

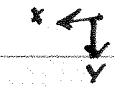
(17)



(A) Net force when @ Top?

↳ F_c





@ Top

$F_T = 2mg$

$$\Sigma F = F_c = F_T + mg$$

$$= 2mg + mg$$

$$\boxed{F_c = 3mg}$$

(B) V_0 @ top Loop?

$$F_c = 3mg$$

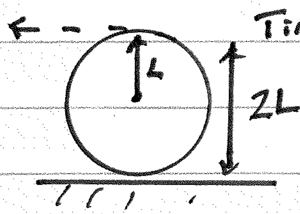
$$F_c = m\frac{V_T^2}{r} \qquad r = L$$

$$\cancel{m}\frac{V_T^2}{L} = 3\cancel{m}g$$

$$V_T^2 = 3Lg$$

$$\boxed{V_T = \sqrt{3Lg}}$$

(C)



Time to Reach ground $V_x \neq 0$, only Y direction

$$\Delta y = V_{oy}t + \frac{1}{2}at^2 \qquad a = g$$

$$2L = 0 + \frac{1}{2}g t^2 \qquad V_{oy} = 0$$

$$\Delta y = 2L$$

$$t^2 = \frac{4L}{g}$$

$$\boxed{t = 2\sqrt{\frac{L}{g}}}$$

(D) Δx $\quad \Delta x = ?$

$$\Delta x = V_{ox}t + \cancel{\frac{1}{2}at^2} \quad \text{NO } a_x = 0$$

$$t = 2\sqrt{\frac{L}{g}} \qquad V_{ox} = \sqrt{3Lg}$$

$$= 2\sqrt{\frac{L}{g}}\left(\sqrt{3Lg}\right)$$

$$\boxed{\Delta x = 2\sqrt{3L^2}}$$

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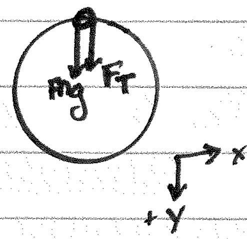
(18) (A)



(B) Given:

$m = 0.50\,kg$

$r = 2m$

$F_T = 20.N$

$V_T = ?$

$F_c = F_T + mg$

$F_c = m\frac{V_T^2}{r}$

$m\frac{V_T^2}{r} = F_T + mg$

$V_T^2 = \frac{r}{m}(F_T + mg)$

$= \frac{2m}{0.50kg}(20.N + 0.5kg(10 m/s^2))$

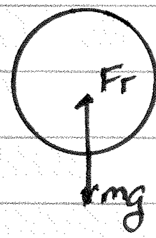
$= 4(25)$

$V_T^2 = 100$

$\boxed{V_T = 10. m/s}$

(B) Tension @ Bottom

V_T = constant



$F_c = F_T - mg$

$m\frac{V_T^2}{r} = F_T - mg$

$F_T = mg + m\frac{V_T^2}{r}$

$= (0.50m)(10. m/s^2) + (0.50kg)\frac{(10. m/s)^2}{2m}$

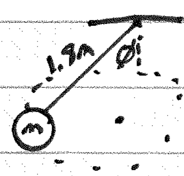
$\boxed{F_T = 30.N}$

(19) Given:

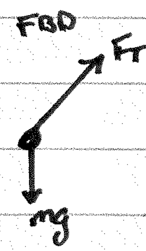
$L = 1.8m$

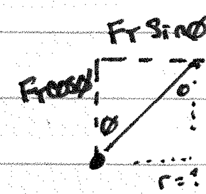
$m = 0.30kg$

$\phi = 30.$



FBD





$\sin\phi = \frac{r}{L}$

$L\sin\phi = r$

$r = 1.8\sin 30^\circ$

$r = 0.90m$

(A) $\Sigma F_y = F_T\cos\phi - mg = m\cancel{a_c}^{0}$

$F_T\cos\phi = mg$

$F_T = \frac{mg}{\cos\phi} = \frac{(0.30kg)(9.8m/s^2)}{\cos 30^\circ}$

$\boxed{F_T = 3.4N}$

(B) $\Sigma F_x = F_c = F_T\sin\phi$

$F_c = m\frac{V_T^2}{r}$

$m\frac{V_T^2}{r} = F_T\sin\phi$

$V_T^2 = \frac{r}{m}F_T\sin\phi$

$= \left(\frac{0.90m}{0.30kg}\right)(3.4N)\sin 30^\circ$

$V_T^2 = 5.1\, m^2/s^2$

$\boxed{V_T = 2.3 m/s}$

(C) $V_T = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{V_T} = \frac{2\pi(0.90m)}{2.3 m/s}$

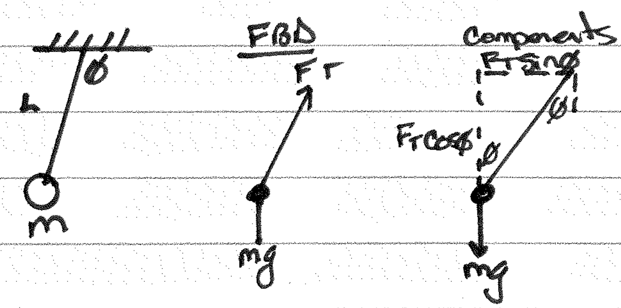
$\boxed{T = 2.5s}$

(20.) **Given:**

$m = 0.10\,kg$ $\quad \phi = ?$

$L = 1.25\,m$ $\quad F_T = ?$

$T = 2.00\,s$



$\sin\phi = \frac{r}{L}$

$$\Sigma F_y = F_T\cos\phi - mg = m\cancel{a}^{0}$$

$$F_T\cos\phi = mg$$

unknown F_T, ϕ

$$\cos\phi = \frac{mg}{F_T}$$

$$= \frac{(0.10\,kg)(9.8\,m/s^2)}{1.2\,N}$$

$$\boxed{\phi = 37^\circ}$$

$$\Sigma F_x = F_c = F_T\sin\phi$$

$$F_c = m\frac{v_T^2}{r}$$

$$m\frac{v_T^2}{r} = F_T\sin\phi$$

$$F_T = \frac{m v_T^2}{r\sin\phi}$$

$$\sin\phi = \frac{r}{L}$$

$$F_T = \frac{m v_T^2 L}{r^2}$$

$$v_T = \frac{2\pi r}{T}$$ * Need T in Given *

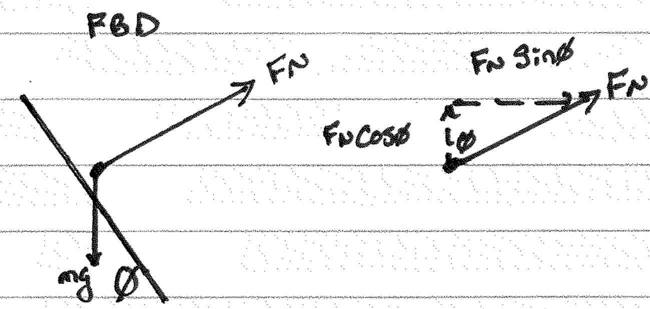
$$F_T = \frac{m 4\pi \cancel{r^2} L}{\cancel{r^2}\, T^2}$$

$$= \frac{m 4\pi L}{T^2} = \frac{(0.10\,kg)4\pi(1.25\,m)}{(2.00\,s)^2}$$

$$\boxed{F_T = 1.2\,N}$$

$\frac{14}{18}$

(21) **Given:**



No f

$y \uparrow \rightarrow x$

$\phi = ?$

Soln:

$$\Sigma F_x = F_N \sin\phi = F_c$$

$$F_c = m\frac{v_T^2}{r}$$

$$F_N \sin\phi = m\frac{v_T^2}{r}$$

$$F_N = \frac{m v_T^2}{r\sin\phi}$$

$$\Sigma F_y = F_N\cos\phi - mg = \cancel{ma}^{\,0}$$

$$F_N\cos\phi = mg$$

$$F_N = \frac{mg}{\cos\phi}$$

$$F_N = F_N$$

$$\frac{\cancel{m} v_T^2}{r\sin\phi} = \frac{\cancel{m}g}{\cos\phi}$$

$$\frac{\sin\phi}{\cos\phi} = \frac{v_T^2}{rg}$$

$$\boxed{\tan\phi = \frac{v_T^2}{gr}}$$

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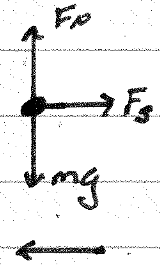
(22.)

(A)



wants to move that way

(B)

$$f_s = F_c$$

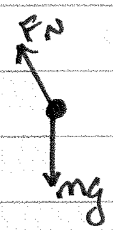
$$u_s F_N = \frac{m v_T^2}{r}$$

$$F_n = mg$$

$$u_s \cancel{m} g = \frac{\cancel{m} v_T^2}{r}$$

$$u_s = \frac{v_T^2}{gr}$$

(C)



(D) F_N must Be greater, Because its Vertical Compent (F_{ny}) is equal to F_g, but its horizontal compent (F_{nx}) is acting Toward the center.

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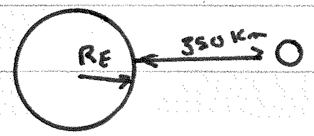
(23) Given:

$V_T = ?$



$r_{Total} = 6.4\times10^6\,m + 350\,km\left(\frac{1000\,m}{1\,km}\right)$
$= 6750000\,m$

Period is speed

$r = r_E + r$

$M_E = 6\times10^{24}\,kg$

$r_E = 6.4\times10^6\,m$

$F_g = F_c$

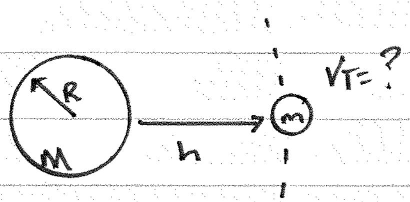
$$\frac{G M_E \cancel{M_S}}{r^{\cancel{2}}} = \frac{\cancel{M_S} V_T^2}{\cancel{r}}$$

$$V_T^2 = \frac{GM_E}{r} = \frac{\left(6.67\times10^{-11}\,\frac{m^3}{\cancel{kg}\,s^2}\right)\left(6\times10^{24}\,\cancel{kg}\right)}{6750000\,\cancel{m}}$$

$= 7699.9$

$$\boxed{V_T = 7700\,m/s}$$

(24) (A)



Sol: $F_{gravity} = F_c$

$$G\frac{M\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}V_T^2}{\cancel{r}}$$

$$\frac{GM}{R+h} = V_T^2 \qquad r = R+h$$

$$V_T = \sqrt{\frac{GM}{R+h}}$$

(B)

i) Astronaut B is correct in saying the acceleration due to gravity above Mars is dependent on the orbital Radius

$\Sigma F = ma \qquad a = g$

$F_{gravity} = mg \qquad g = \frac{GM}{r^2}$

$\frac{GM\cancel{m}}{r^2} = \cancel{m}g$

ii) Astronaut A is incorrect, The acceleration is dependent on the Mass of the Planet, rather than the spaceship

(C) $V_T = ?$

$V = \frac{D}{T}$

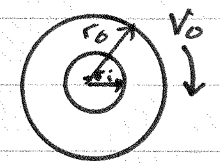
$D = 2\pi r$

$\therefore V = \frac{2\pi r}{T} \qquad r = R+h$

$$\boxed{V = \frac{2\pi(R+h)}{T}}$$

$\frac{17}{18}$

25 A



$V_o = ?$

Given:

r_o g on Earth

r_i $g/2$

$F_{net} = F_c$

$F_N = m\frac{V_o^2}{r_o}$

$F_n = mg$

$\cancel{m}g = \cancel{m}\frac{V_o^2}{r_o}$

$$\boxed{V_o = \sqrt{g r_o}}$$

B

$T = ?$

$V = \frac{D}{T}$

$D = 2\pi r_o$

$T = \frac{2\pi r_o}{V_o}$ $\quad V_o = \sqrt{g r_o}$

$$\boxed{T = \frac{2\pi r_o}{\sqrt{g r_o}}}$$

C r_i in terms of R_o?

Solve for T. The Period of Rotation is the same for Both Eqn as the spacecraft is on disc, & All points on the disc rotate with same Angular Velocity

r_i @ $\frac{g}{2}$

$T_{r_o} = \frac{2\pi r_o}{\sqrt{g r_o}}$ $\qquad T_{r_i} = \frac{2\pi r_i}{\sqrt{\frac{g}{2} r_i}}$ $\quad \frac{g}{2}$

$(\sqrt{g r_o})^2 = \frac{4\pi^2 r_o^2}{T_{r_o}^2}$ $\qquad \left(\sqrt{\frac{g}{2} r_i}\right)^2 = \frac{4\pi^2 r_i^2}{T_{r_i}^2}$

$T_{r_o}^2 = \frac{4\pi^2 r_o^{\cancel{2}}}{g \cancel{r_o}}$ $\qquad T_{r_i}^2 = \frac{(4\pi^2 r_i^{\cancel{2}})2}{g \cancel{r_i}}$

$T_{r_o}^2 = T_{r_i}^2$ $\qquad$ * Same Period of Rotation!!

$\frac{\cancel{4\pi^2} r_o}{\cancel{g}} = \frac{(\cancel{4\pi^2} r_i)2}{\cancel{g}}$

$$\boxed{r_i = \frac{r_o}{2}}$$

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(25) Conti

d. Stays the same. Newton's first law states that in the absence of a net external force, an object will stay in motion. In this case, it is rotational motion, and in the absence of an external torque, it will keep rotating at the same rate started by the force/torque exerted by the thrusters.

A satisfactory answer could also be that the rotational speed could decrease based on the small amount of atmosphere at the satellite's orbit that would decrease its rotation rate. Also, the gravitational pull on the satellite from the plan could decrease its rotational rate depending on its orientation in space.