AP Physics 1 Unit 6 – Rotation

6.1– Rotational Kinematics

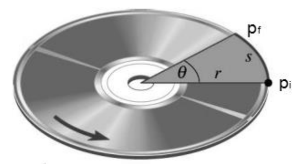
Focus Question: How is rotational kinematics different from linear kinematics?

Review: *Kinematics -*

Revolution –

ex: a roller coaster going around a loop, Earth orbiting the sun.

Rotation -

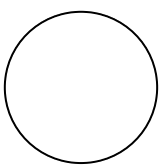


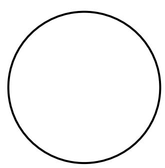
ex: a wheel's spin, the rotation of Earth about its axis.

Describe kinematic quantities in rotational motion and relate them to linear motion.

Angular Displacement - Vector quantity representing how much a rotating object spins (the angle)

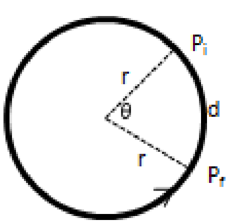
At time t = 0s. After a time, t





Units of angular displacement:

Relationship between linear distance and angular displacement:



Example A: A hipster spins a vinyl copy of an album you've probably never heard of on their vintage record player. The record spins 10 times.

a) Find the angular displacement of the record in that time.

b) While the record was spinning, a fly stands on the record 10 cm from the center of the disc. Find the length of the fly's path while the record is spinning.

Angular Velocity - Rate of angular displacement (how fast an object is spinning)

Angular velocity: $\omega = \frac{angular\ displacement}{time}$

Units of angular velocity:

Vector direction (Right-hand rule): Curl the fingers of your right hand in the direction of motion, the direction of the vector is the direction your thumb points.

Relationship between linear speed and angular velocity- divide both sides of the relationship between linear displacement and angular displacement by time:

Angular Acceleration - Rate of change of angular velocity. For an object rotating in the positive direction, positive angular acceleration means it is spinning faster and faster and negative acceleration means the object's spin is slowing down.

Units of angular acceleration:

Relationship between linear speed and angular velocity-

Kinematics Equations

Linear Equations:

$$d = v_i t + \frac{1}{2} a t^2$$

$$d = v_f t - \frac{1}{2} a t^2$$

$$v_f^2 = v_i^2 + 2ad$$

$$v_f = v_i + at$$

$$d = \left(\frac{v_i + v_f}{2}\right) t$$

Rotational Equations:

Apply the kinematics equations to rotational motion.

Example B: A wheel making 800 rev/min slows down to 500 rev/min while making 60 revolutions.

a) What is the angular acceleration?

b) How long does it take the wheel to slow down?

c) Find the linear speed of an object .35 m from the center of the wheel when the wheel is spinning at 500 rev/min.

Example C: The angular acceleration of a wheel is 4.0 rad/s^2. How many revolutions does the wheel make in the first 6.0 seconds if it is initially at rest?

Example D: A wheel is rotating at 4.0 rad/s. Its rotation accelerates at a rate of .25 rad/s^2.

a) What is the angular displacement in radians during the first 3.0 seconds?

b) What is the angular velocity at the end of 3.0 seconds?

c) What is the linear distance traveled by an object .40 m from the center of the wheel?

6.2– Equilibrium of Rigid Bodies (Torque)

Focus Question: When is in object in static equilibrium?

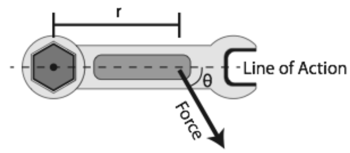
- **Rigid Body -**

For a rigid body to be in static equilibrium, the object must be in both ______________ equilibrium and ___________________ equilibrium.

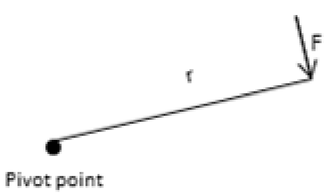
- Translational Equilibrium –
- Rotational Equilibrium -

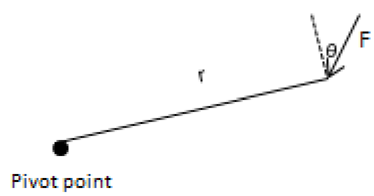
Calculate torque acting on an object.

- Torque –



- Magnitude of torque –

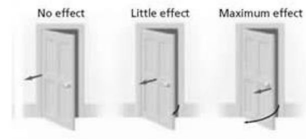




*only the component of forces perpendicular to the axis of rotation causes rotation.

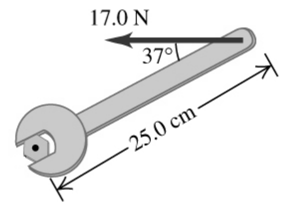
- Direction – Torque is a vector quantity, which means it has direction and magnitude. The directions of torque are clockwise or counter wise.

- Torque/distance relation: A force applied further away from the center of rotation creates a greater torque.
  ex: It's easier to open an open by pushing on the side opposite the hinges rather than near the hinges of the door.

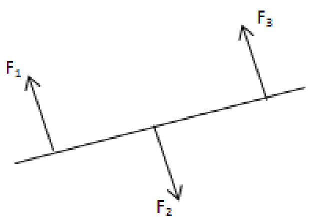


- Units of torque:

Example A: Bob the Builder uses a wrench to tighten a bolt as shown. He applies a 17.0 N force 25 cm from the bolt an angle of 37° with the axis of rotation. Calculate the magnitude and direction of the torque on the bolt.



- Pivot point - Some objects have a natural pivot point, but any point can be selected as a pivot point. The value of the torque will depend on the which point is the pivot. If there is no natural pivot point, it's best to choose:
  a) one of the ends b) a point where a force is applied

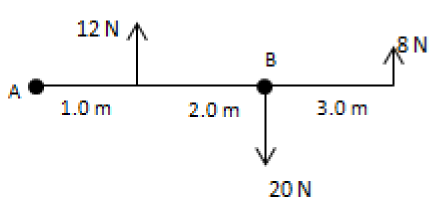


Rotational Equilibrium

Sum of clockwise torque = Sum of counterclockwise torque

If the clockwise torque and counterclockwise torque are the same about a pivot point (any pivot point can be chosen), an object will not rotate about the pivot point and is in rotational equilibrium.

Example B: Check if the following object is in rotational and translational equilibrium:



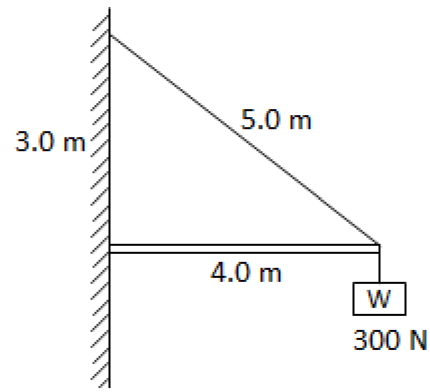
Example C: A uniform meter stick weighs 25.0 N. An 80.0 N weight is hung at the 20.0 cm mark and a 70.0 N weight is hung at the 90.0 cm mark.

a) What is the magnitude of the upward force needed to balance the meter stick?

b) Where should the force be applied so that the stick hangs horizontally?

Example D: A 800 N painter stands 3.00 m from the left end of a scaffold that is 4.00 m long. The uniform scaffold weighs 100 N and is hung by a chain at each end. What is the tension in each chain?

Example E: The horizontal strut in the figure to the right is uniform and weights 100 N. Find a) the tension in the cable. b) the normal force of the wall on the strut.



6.3– Newton's 2nd Law for Rotation

Focus Question: What causes rotational motion?

In linear motion, acceleration is caused by force (Newton's second law). In rotary motion, angular acceleration is caused by torque (T).

Linear motion: Rotary motion:

Calculate moment of inertia.

Only _________ resists acceleration in linear motion. In angular motion, both the _______ and _______ of an object resist angular acceleration. A quantity called ___________________________ is a property of both mass and shape and is a measure of an object's ability to resist rotation.

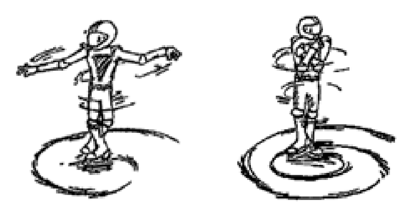
Rotational inertia is analogous to mass, but more complex. Rotational mass is based on the mass of a body and the distance of the mass from the axis of rotation.

Take 2 rods: Rod 1  Rod 2 

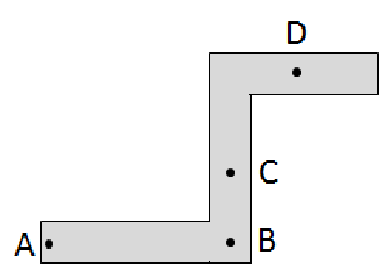
If a mass was placed on the end of each rod, and force was applied to one end, the mass at the end of rod 2 would cover a greater distance. . However, it would have a larger circumference to travel around, so the angular displacement would not be as large as for the smaller rod.

Formulas for rotational inertia are derived in calculus. In Physics 1, the formula will always be provided. However, you are expected to know qualitatively which objects have more inertia (more resistance to rotation).

Example A: An Olympic ice skater is spinning and pulls her arms in to speed up her spin. Explain how this is possible with physics.



Example B: The thingamajic shown can be rotated around anyone of 4 pivot points shown. Rank the moment of inertia of the object about each pivot point.



Apply Newton's 2^{nd} Law for rotation.

Example C: A tennis ball has a mass of .32 kg and a radius of .056 m. The rotational inertia of the ball is given by $I = \frac{2}{5}mr^2$, where r is the radius of the ball. What torque is required to give the ball an angular velocity of 5.0 rad/sec in .60 seconds?

Example D: A rod is hinged on its left side. The uniform rod is 2.0 m long and has a mass of 3.6 kg. What is the angular acceleration of the rod at the instant is released from a horizontal position?

6.4– Conservation of Angular Momentum

Focus Question: What is angular momentum?

Define momentum in angular motion.

- Angular momentum is similar to linear momentum.
  Angular Momentum

  *units -
- Angular Impulse

Example A: A uniform 3 kg rod is 2 m long and free to rotate about its center. By the hammer of Thor, the rod is struck by a 1500 N force for .08 s directly perpendicular to the rod. What is the angular velocity of the rod after impact? (I for rod pivoted at center = $\frac{1}{12}ML^2$)

Apply conservation of angular momentum to solve problems.

In an angular “collision”, angular momentum is always conserved unless external torques act on the system.

*internal torques do not change angular momentum of a system.

Example B: A baggage carousel has a mass of M=500 kg and a is in the shape of a disk with r = 2.0 m. It rotates at 1.0 rad/s when ten pieces of luggage with average mass of m = 20 kg are dropped on the carousel. Find the speed of the carousel after the luggage is added if no external torques act on the carousel.

- The angular momentum, $\vec{L}$, of a particle about an axis perpendicular to the plane of the particles motion:

Example C: A bullet of mass m = .20 kg is shot at a 40 kg door at its unhinged end. The speed of the bullet is initially 200 m/s and travel perpendicular to the plane of the door. The bullet becomes embedded in the door, which is 2.5 m high and 1.5 m wide.

a) What is the angular velocity of the door after the door after the bullet strikes?

*The door is a rod hinged at one end (I= $\frac{1}{3}ML^2$). The height is negligible.

b) How would the answer change if the bullet hit closer to the door?

6.5– Work & Energy in Rotational Motion

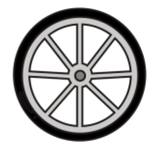
Focus Question: How do we deal with a system with translational and rotational motion?

Solve for rotational kinetic energy.

Kinetic energy of rotation –

Analyze systems with moving axis of rotation.

- Often times, a body has both translational and rotational motion, such as the wheel on a car.
- Newton's Laws for rotational mechanics still remain valid as long as:
  -
  -
- Total kinetic energy of a rigid body -

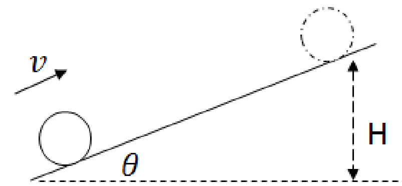


Example A: A soccer ball of mass M and radius R is rolling at a speed V. Find the total kinetic energy. The moment of inertia for a solid sphere is $\frac{2}{5}MR^2$.

*Total kinetic energy does *not* depend on radius*

Example B: A yo-yo is a solid disk or radius R and mass M. The yo-yo is released from rest while the free end of the yo-yo string is held stationary. Find a) the acceleration of the yo-yo and b) the tension in the yo-yo string.

Example C: A solid wheel of mass M and radius R moves up an incline as shown. The speed of the ring is v as it enters the incline. Find how up the incline the wheel goes if



a) The wheel slips while going up the incline. (frictionless)

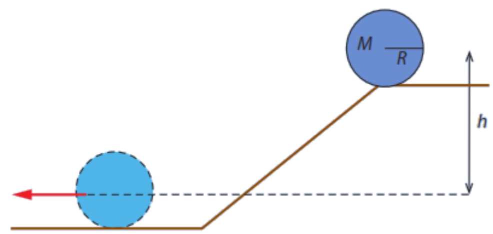
b) There is no slipping of the wheel as it moves up the incline. (friction is present)

c) How would the answer to b) change if the wheel were hollow?

Background When friction is present, it is used to convert rotational kinetic energy into linear kinetic energy. The magnitude of the negative rotational work done by friction equals the positive linear work done by friction. Thus friction does no net-work on the wheel.

NOTE In the no friction case (slipping), only translational kinetic energy is converted into gravitational potential energy. In the case with friction (no slip), more kinetic energy is converted into gravitational potential energy and the wheel will rise more. The torque causes the object to slow down is provided by the parallel component of weight. Since friction causes the angular acceleration to be less negative tan the slipping case, **friction points in the positive direction** for this problem.

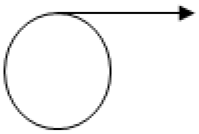
Example D; Rolling down an incline: A wheel of radius M and radius R rolls without slipping down an incline.



a) Find the linear speed of the center of mass of the wheel when it reaches the bottom of the incline.
b) Calculate the acceleration of the wheel down the incline.
c) Determine the minimum coefficient of friction required to prevent slipping down the incline.

Solve for work and power in rotational motion.

A force applied to a rotating body does work on the body. The work can be expressed with torque and angular displacement:



*only the component of the force perpendicular to the axis of rotation does work.

Example D: A golf cart wheel has a radius of R and a mass of M. The wheel is brought to rest from an initial angular speed of ω. The wheel turns through an angle of θ before stopping.

a) What is the work done to slow the wheel?
b) What torque is required to slow the wheel?
c) What is the power required to stop the cart?