ROTATIONAL INTERTIA AND ROTATIONAL KINETIC ENERGY

LEARNING TARGET

- **Explain rotational Inertia of a system and how to use it to calculate the rotational kinetic energy of that system**
- **Learn the technique for finding rotational inertia of a rigid body, including using of the parallel-axis theorem**
- **Predict the behavior of rotational collision situations by the same processes that are used to analyze linear collision situations using an analogy between impulse and change of linear momentum and angular impulse and change of angular momentum**

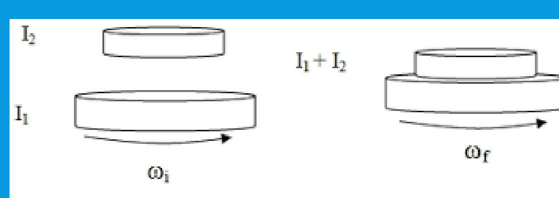
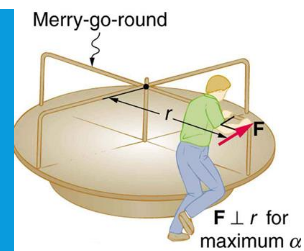
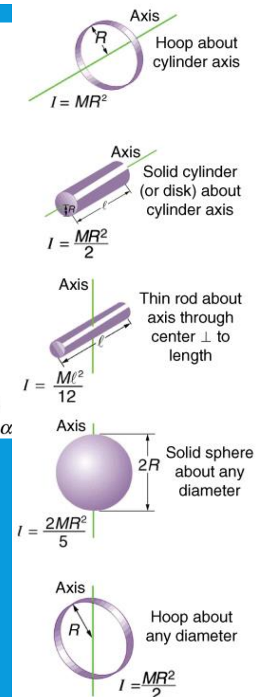


Figure 2 *Dropping a stationary disk onto a rotating disk*

ROTATIONAL INERTIA

- **Rotational inertia** is the rotational equivalent of mass (inertia) in linear motion
- It represents how hard it is to change the rotation of an object. The greater the rotational inertia, the greater the torque required for an equal angular acceleration
- **The rotational inertia** of a rigid body depends on the rigid body's **mass** and how that mass is **distributed** relative to the rigid body's rotational axis



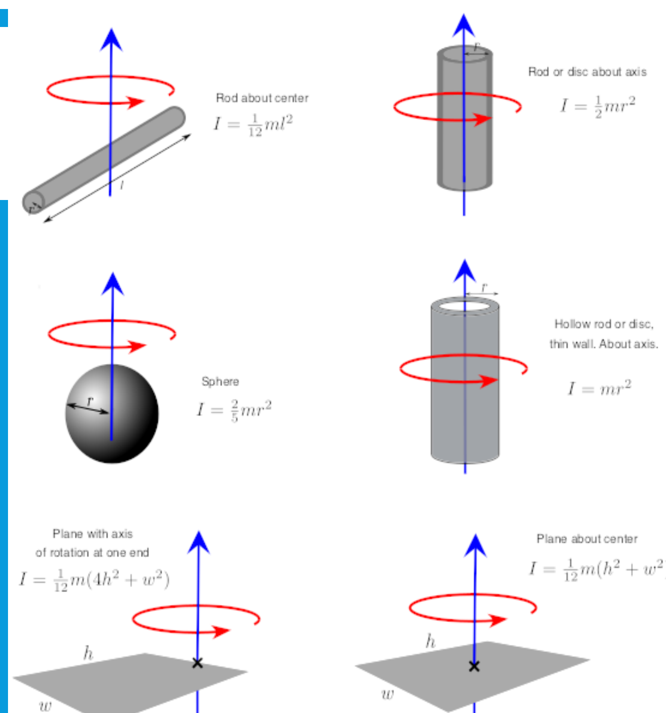


MOMENT OF INERTIA

Rotational inertia depends on the mass and shape (distribution of mass) of an object

- $I = kmr^2$ where k is the shape constant
- You do not need to memorize (eqn) various rotational inertias, but you do need to be able to qualitatively know which objects have greater/smaller rotational inertia

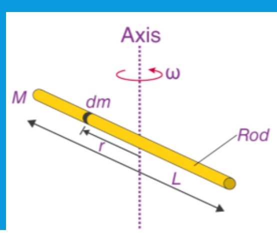
$$I_{hoop} > I_{disk} > I_{solid\ sphere}$$

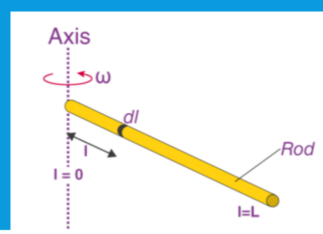


ROTATIONAL INERTIA

Three important properties of rotational inertia

1. **Rotational inertia is additive**
2. **The farther away from the rotational axis is a rigid body's mass lies, the greater the rigid body's inertia**
3. **The rotational inertial depends on the rotational axis.**
   a) **If a different axis is used, then the rotational inertia changes also.**





ROTATIONAL ENERGY

Rotating requires kinetic energy; specifically:

$$K_r = \tfrac{1}{2} I\omega^2$$

Since K_r depends on I, how much energy is used for rotation depends not just on the total mass, but on the distribution of that mass.

For instance, for a mass rolling down a ramp where total energy is conserved:

$$(U_g + K_r + K_t)_{initial} = (U_g + K_r + K_t)_{final}$$

A greater I value means more energy is used for rotation, so less is available for translation. **(THIS IS KEY TO REMEMBER!!!)**

EXAMPLE - TURBINE KINETIC ENERGY

The blades of a wind turbine have a combined rotational inertia of 2.00 x 10^2 kg m^2 for rotation around the turbine axis. If the blades make a 4.00 complete revolutions every minute, what is the rotational kinetic energy of the blades?

Rotational Kinetic Energy

$K_r = \frac{1}{2} I \omega^2$

Angular Velocity

$\omega = \frac{\Delta \emptyset}{\Delta t}$

Convert into rad/s

$$\omega = \frac{\Delta \emptyset}{\Delta t} = \left(\frac{4.0\ rev}{min}\right)\left(\frac{2\pi}{1\ rev}\right)\left(\frac{1\ min}{60\ sec}\right) = 0.419\ rad/s$$

$$K_r = \frac{1}{2} I \omega^2 = \frac{1}{2}\ (2.00 \times 10^2\ kg\ m^2)\ (0.419\ rad/s)^2$$

$$K_r = 175\ kg\ m^2/s^2 = 175 J$$

ROTATIONAL MOMENTUM

- Objects that are rotating have both angular momentum and rotational kinetic energy. Objects that are translating can also have angular momentum about some point.
- The angular momentum of a system will stay constant unless a net torque acts on the system.
- **When to use**
  - Collisions in rotation
  - Finding velocity of object rolling on a ramp or spinning
  - **Common Examples**
    - Collisions in rotation
    - Objects rolling with friction
    - Objects in elliptical orbit

ANGULAR MOMENTUM

The angular momentum, **L**, of a system is defined as:

$$\mathbf{L} = \mathbf{I}\boldsymbol{\omega}$$

Angular momentum is conserved: If there is no net external torque on a system, then:

$$\mathbf{L_i} = \mathbf{L_f}$$

Consider a gymnast doing a flip. When they push off the ground, they create a torque and give themselves angular momentum. In the air, they tuck - decreasing I and (since L is conserved) increasing ω. To land, they do the reverse, slowing their rotation.

ANGULAR MOMENTUM AND TORQUE

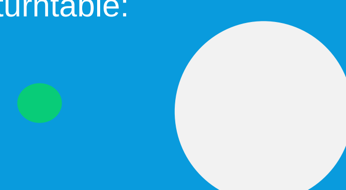
As long as net torque on the system is zero, angular momentum is conserved.

However, if there is an external net torque, it provides an angular impulse (like how an external net force provided a linear impulse).

That angular impulse is equal to the change in angular momentum:

$$\tau \Delta t = \Delta L$$

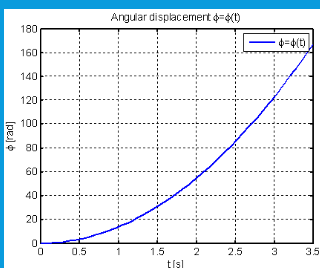
Objects moving linearly can also have angular momentum if they're moving tangential to a pivot point. For instance, consider catching a ball while standing on the turntable:



If the ball is caught near the edge of the turntable, it'd transfer both linear and angular momentum when caught.

GRAPHS

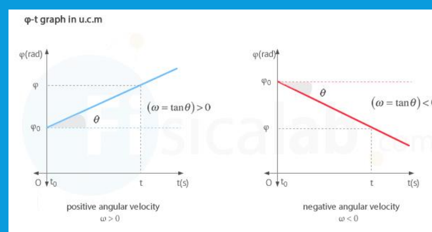
Angular Displacement vs Time

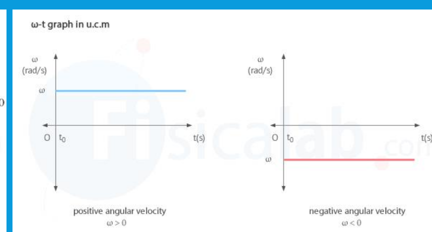


Slope = $\frac{\emptyset}{t}$= ω Angular Velocity

Area under graph = Øt = nothing

Angular Velocity vs Time



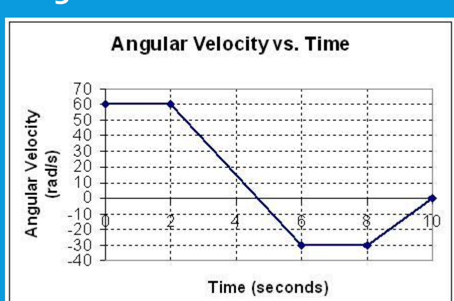


Slope = $\frac{\omega}{t}$= α Angular Acceleration

Area under graph = ωt = ØΔ Angular displacment

GRAPHS

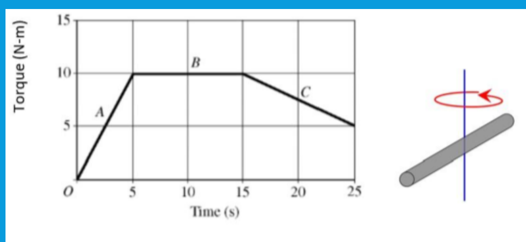
Angular Acceleration vs Time



Slope

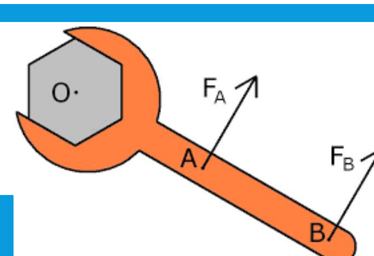
Area under graph = α Δt = Δω angular speed

GRAPH - TORQUE



Slope = Area under graph = τ t = Δ L angular impulse

TORQUE



A force (or component) directed tangential to a pivot point (fulcrum, hinge, etc.) causes a torque. How much torque is given by:

$$\tau = rF\sin\theta$$

Where r is the distance from the pivot point to where the force is applied, F is the amount of force, and θ is the angle between r and F.

Torque is a vector - its direction is along the chosen axis of rotation, and can be determined by a cross product (vector product):

$$(\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F})$$

We can determine the direction using a right hand rule (next slide)

CALCULATING TORQUE

- Read problem and draw a rough figure.
- Extend line of action of the force.
- Draw and label moment arm.
- Calculate the moment arm if necessary.
- Apply definition of torque:

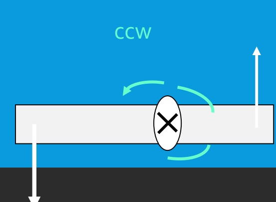
$$\boxed{\tau = Fr}$$

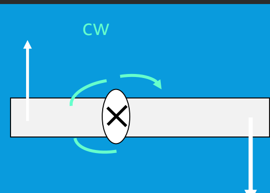
Torque = force x moment arm

SIGN CONVENTION FOR TORQUE

By convention, counterclockwise torques are positive and clockwise torques are negative. (use Right hand rule)

Positive torque: Counter-clockwise, out of page

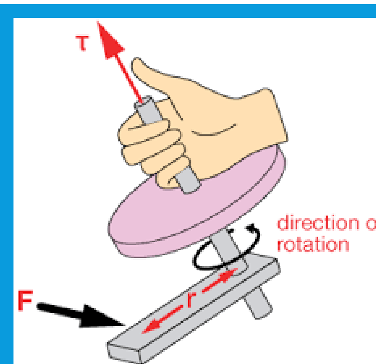




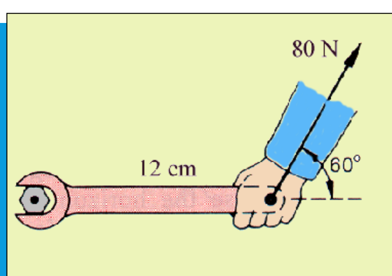
Negative torque: clockwise, into page

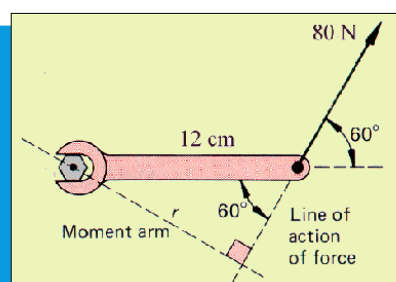
TORQUE RIGHT-HAND RULE

1. Use your **right** hand (see name of rule)
1. Point fingers in the direction of **r** (from pivot point toward **F**)
1. Curl fingers in the direction **F** points
1. The direction your thumb points is the direction of **τ**



EXAMPLE 1: AN 80-N FORCE ACTS AT THE END OF A 12-CM WRENCH AS SHOWN. FIND THE TORQUE.





- Extend line of action, draw, calculate r.

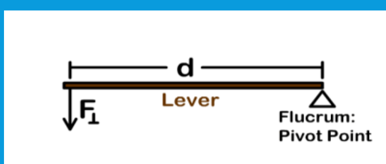
$r = 12\text{ cm}\sin 60^0$
$= 10.4\text{ cm}$

$\tau = (80\text{ N})(0.104\text{ m}) =$
8.31 N m

FORCES CAUSING ROTATION

We know from Unit 2 that a net force acting on an object will cause that object (specifically its center of mass) to accelerate.

Depending on how/where that force is directed, it can also cause that object to rotate:



Consider our meter stick and masses : Each hanging mass applied a gravitational force to the meter stick; if that force was not at the fulcrum, it caused a rotation about the fulcrum. A force causing a rotation provides a torque.

Torque (τ) is the rotational equivalent of force; forces cause linear acceleration, while torques cause angular acceleration

EQUILIBRIUM

In order to be in **mechanical equilibrium**, an object/system must satisfy two conditions:

1. The net force on the object/system must be zero

1. The net torque on the object/system must be zero

An object in equilibrium must have a constant translational velocity (which may or may not be zero) and a constant angular velocity (which may or may not be zero). Stationary objects are a good example of objects in equilibrium.

TORQUE

τ = rF = mra

To relate torque to angular acceleration, we can use the relationship between angular and linear acceleration, a/r = α. Thus:

τ = m$r^2\alpha$

However, this is just torque for one particle on the object

particles on the object:

$\sum\tau = \sum mr^2\alpha$

Since the whole object rotates together, α is the same for all particles - m and r are the values that may change. We define the **moment of inertia**, $I = \sum mr^2$. This is our "rotational mass" equivalent. Our 2nd law for rotation is:

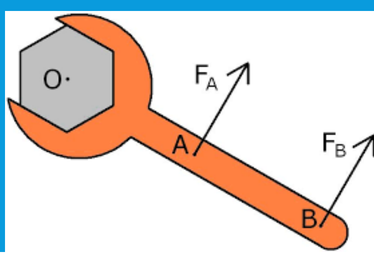
$\sum\tau = I\alpha$

REVIEW

TORQUE REVIEW

A force (or component) directed tangential to a pivot point (fulcrum, hinge, etc.) causes a torque. How much torque is given by:

$$\tau = rF\sin\theta$$



Remember that, just like we have Newton's 2nd Law for forces:

$$\Sigma F = ma$$

There is an equivalent version for torques:

$$\sum \tau = I\alpha$$

Where I is the **moment of inertia** of that object. If we need I, it is usually given (or we can look up common values).

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BIG IDEAS TO REMEMBER

Circular Motion:
Circular motion requires a centripetal (toward the center) acceleration from a centripetal net force.

Gravitation: There's a law for that. Use the equation.

Energy: Total energy is conserved for closed, isolated systems. Work done on a system changes the energy of the system

Momentum: Momentum is conserved as long as there is no external net force. Remember the difference between elastic/inelastic

Simple Harmonic Motion:
Harmonic motion repeats with a specific period. Makes a sine wave.

Torque and Rotation: There's a rotational version of nearly everything we saw before: kinematics, torque, angular velocity, rotational energy, etc.

BIG IDEAS TO CONSIDER

Conservation Laws:
Is something conserved? Total energy, kinetic energy, linear momentum, angular momentum? If so, write an "initial = final" equation

Static vs. Dynamic:

Is our object stationary? If so, net torque and net force must be zero.

If it has angular or linear acceleration, use the 2nd law for torque or for force. Either way, write out the equation and sub in all the forces/torques