Circular Motion

AP Physics Unit 3

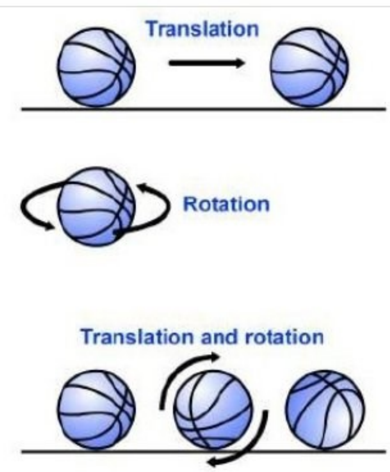
Main Ideas - Circular motion & Gravitation

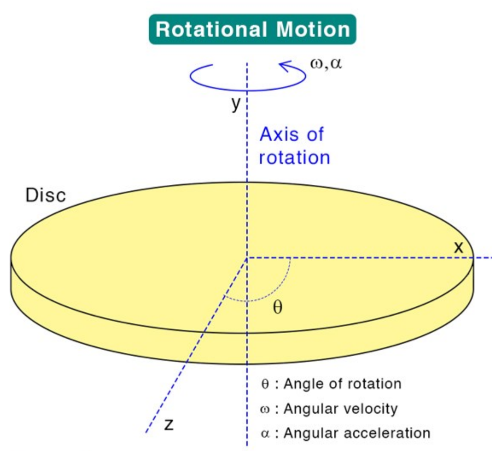
- **Gravitation** is a force of universal importance; add circular motion and you starting explaining the motion of the planets
- An object moving in a circle is accelerating even if its speed is constant
- An object in **uniform circular motion**, the net force exerted on the object points toward the center of the circle
- **Newton's law of universal gravitation** explains the orbit of the Moon, and introduces us to the concept of field.
- Newton's law of universal gravitation begins to explain the orbits of planets and satellites.
- Apparent weight and what it means to be "weightless"

Background information
Rotational Vs Translational Motion

- **Rotational** and **Translational** motion can be analyzed separately.
  - **Translational motion** is movement that changes the **position** of an object, as opposed to **rotation**
  - **Example:** when a bowling ball strikes the pins, the pins may spin in the air as they fly backward.
    - These pins have both **rotational & translational motion**.
- We are going to isolate
  - Rotational motion.
  - Explore how to measure **the ability of a force to rotate an object**

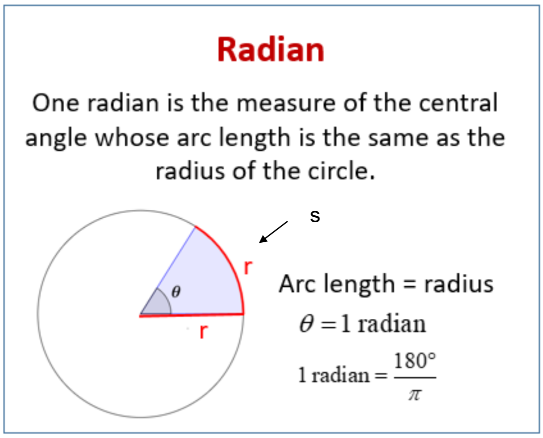
Background information
Rotational Vs Translational Motion





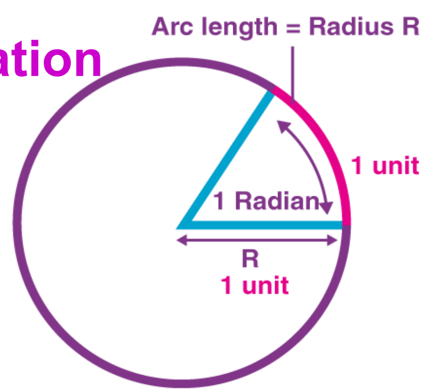
Background information
Radian and Degrees

- In degrees, once around a circle is 360º
- In Radians, once around a circle is 2¶



S = Arc length

Background information
Radian “unitless” defined



- Note that “**Radian**” is not a unit. Radian is ratio that describes an angles the ratio of the arc length to the radius. This ratio is dimensionless (has no unit), because the units cancel.

This means that an angle described in radian has no unit, and therefore never needs to be converted from one unit to another. However, we often write “rad” after an angle is measured in radians to remind ourselves that the quantity describes an angle.

Background information

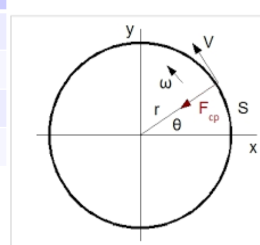
Variables – Linear and Angular

- **S = Arc length**
  - $s = r\theta$
  - units – m
- **Ø= rotation/Angular displacement**
  - Units – Radians
- **r = radius**
  - units – m
- **ω = angular speed/Velocity**
  - $\omega = \frac{\Delta \emptyset}{\Delta t}$
  - Units – rad/sec
- **Tangential Velocity**
  - $V_T = \frac{\Delta s}{\Delta t} = \frac{r \Delta \emptyset}{\Delta t} = r\,\omega$
  - Units – rpm or Rev/Sec
- **a_c = Centripetal acceleration**
  - $a_c = \frac{v^2}{r} = r\omega^2$
  - Units – m/s^2
- **Tc = tangential acceleration**
  - $a_T = r\alpha$
  - Units – m/s^2
- **α = angular acceleration**
  - $\alpha = \frac{\Delta \omega}{\Delta t}$
  - Units – rad/s^2

Comparing Kinematic Variables

- Depending on the book/professor/editor there are different variables for angular

	Translational (linear)	Angular
position	X	Ø
Displacement	**s,** d, Δx	ΔØ
Velocity/Speed	v	ω (omega)
Acceleration	a	α (alpha)



S = arc length, m

Ø =Rotational displacement, rad

ω = angular velocity, rad/s

α = angular acceleration also called Rotational acceleration, $1/s^2$, rad/s^2

Kinematic Equation Parallels

Translational	Rotational
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$\Delta x = v_0 t + \frac{1}{2} a t^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

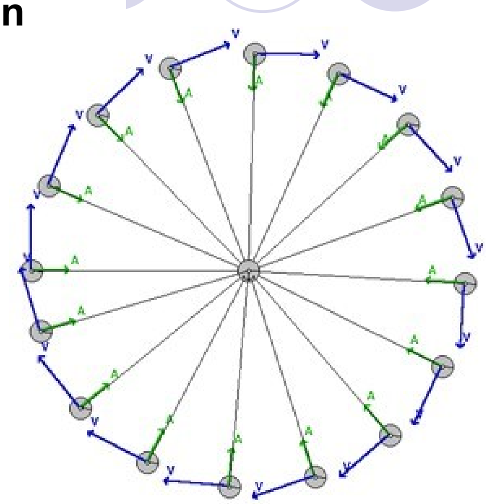
Analyzing motion in a circle

Uniform Circular Motion

Uniform circular motion – An object going around a circular path at a **constant speed.**

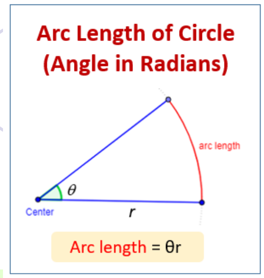
A moon or planet in orbit is (very nearly) an example of **Uniform Circular Motion.**

It's called **"uniform"** because the object moves in a circle at a constant speed - it doesn't speed up or slow down, just changes direction



Analyzing motion in a circle

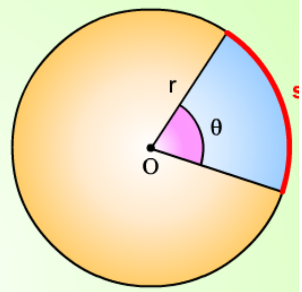
Arc Length



Measurement of arc length in Radian

In general, if the angle of a sector, θ, is measured in degree,

then the **length of its arc**, $s = \frac{\theta}{360} \times 2\pi r$



If θ is measured in **radians**,

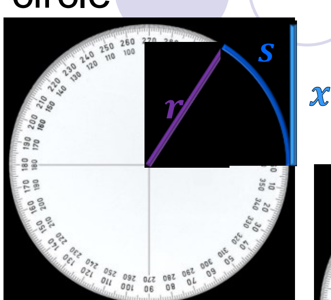
then the length of its arc, $s = \frac{\theta}{2\pi} \times 2\pi r$ $2\pi \text{ rad} = 360°$

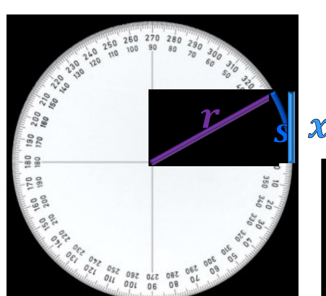
$\therefore$ $s = r\theta$

Analyzing motion in a circle

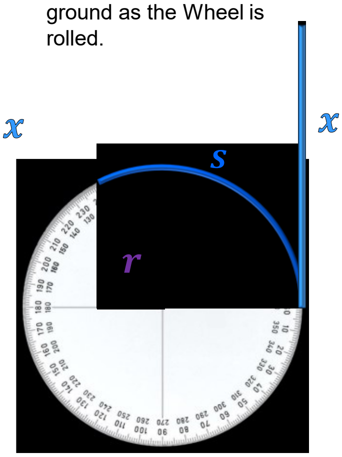
Arc Distance s

Tangential Distance, x





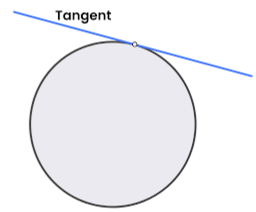
Think of the tangential distance is the distance a wheel covers on the ground as the Wheel is rolled.



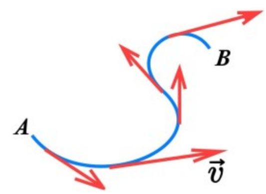
$$s = r\theta$$

Analyzing motion in a circle

- A tangent is simply a line that touches a function at only a single point



- **Tangential velocity** is the component of motion along the edge of a circle measured at any arbitrary instant.
  - As the name suggests, tangential velocity describes the motion of an object along the edge of this circle whose direction at any given point on the circle is always along the tangent to that point.

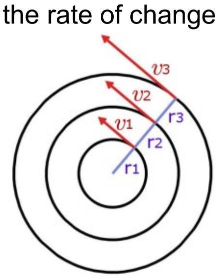


Analyzing motion in a circle

Uniform Circular Motion

What determines the amount of acceleration needed to cause uniform circular motion?

1. Tangential Speed: If two objects move in identical circles, one fast and one slow, which one must accelerate more? (Remember, acceleration is the rate of change of velocity)



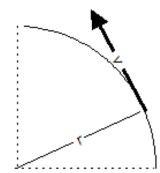
Note:
Notice how the V_t Vector increase as the r length increases

2. Radius: If two objects move in circles at the same speed, one with a large radius and one with a small radius, which one must accelerate more?

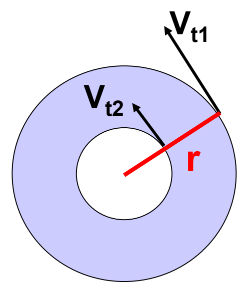
Circular motion deals with the change of direction of an objects velocity

Tangential Velocity

- The **tangential speed (v_t)**
  - Used to describe the speed of an object in circular motion
  - is the object's speed along an imaginary line drawn tangent to the circular path.

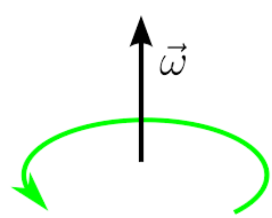


- Tangential speed depends on the **distance (r)** from the object to the center of the circular path
  - Which point is moving faster outside or inside of the circle?
    - V_{t1} has to cover more distance, so it has a great tangential speed

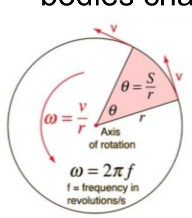


- When the tangential speed is constant, the motion is described as **uniform circular motion.**

Angular velocity

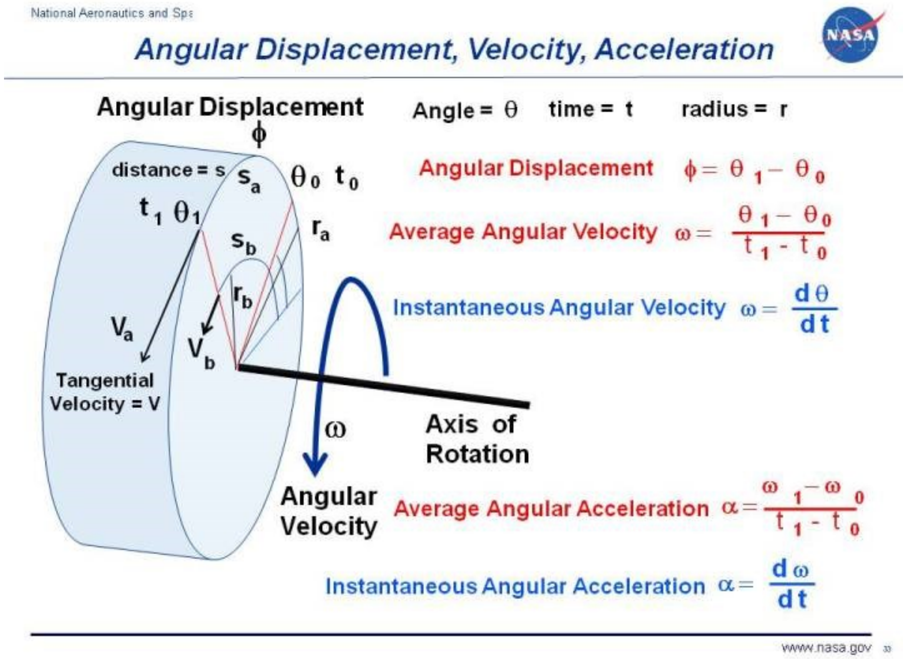


- **Angular velocity,**
  - Also called rotational velocity,
  - Measure of how fast the central angle **,Ø,** is changing over time
  - Or revolves, about an axis
  - Or at which the angular displacement, **Ø,** between two bodies changes

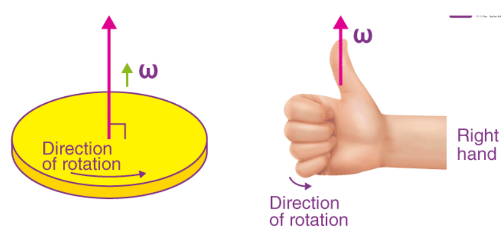


$$\omega = \frac{\Delta\theta}{\Delta t}$$

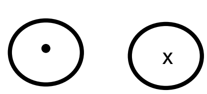
notice, **r**, is not a variable. Therefore angular velocity is the same throughout the circle



Angular velocity



Angular velocity ω is a vector, so we must include magnitude and direction. The direction of the angular velocity is along the axis of rotation, and points away from you for an object rotating clockwise, and toward you for an object rotating counterclockwise.

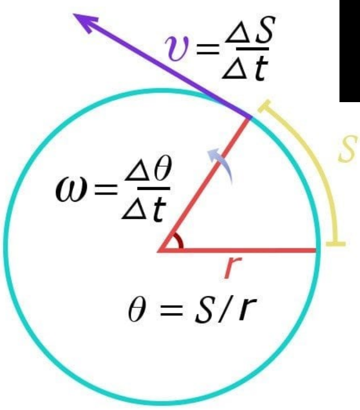


Dot – Vector coming upward
x – Vector going down
Think of an arrow

The **direction of angular velocity is determined by right hand rule.**

- If you hold the axis with your right hand and rotate the fingers in the direction of motion of the rotating body then thumb will point the direction of the angular velocity.

Linear Velocity vs Angular Velocity



What minimum **angular velocity** is necessary to keep these people from sliding down the wall?

$r = 2.5\ m$

$\mu_s = 0.75$

$m = m \qquad \omega = ?$

$$F_f = F_g$$

$$\mu_s |\vec{F}_N| = mg$$

$$\mu_s m r \omega^2 = m g$$

$$\omega^2 = \frac{g}{\mu_s r}$$

$$\omega = \sqrt{\frac{g}{\mu_s r}}$$

$$\omega = \sqrt{\frac{9.8}{0.75 \cdot 2.5}}$$

$$\omega = 2.3\ rad/s$$

Double the Radius

ΔTangential Velocity? **Result 2x V_t** $v_t = r\omega$

Δ Angular Velocity? **Result Same ω** $\omega = \frac{\Delta\phi}{\Delta t}$