Problem | A

HOOKE'S LAW

PROBLEM

The pygmy shrew has an average mass of 2.0 g. If 49 of these shrews are placed on a spring scale with a spring constant of 24 N/m, what is the spring's displacement?

SOLUTION

1. DEFINE

Given:

$$m = \text{mass of one shrew} = 2.0 \text{ g} = 2.0 \times 10^{-3} \text{ kg}$$

 $g = 9.81 \text{ m/s}^2$

k = 24 N/m

Unknown:

2. PLAN

Choose the equation(s) or situation: When the shrews are attached to the spring, the equilibrium position changes. At the new equilibrium position, the net force acting on the shrews is zero. So the spring force (given by Hooke's law) must be equal to and opposite the weight of the shrews.

$$\mathbf{F_{net}} = \mathbf{0} = \mathbf{F_{elastic}} + \mathbf{F_{g}}$$

$$F_{elastic} = -kx$$

$$F_g = -m_{tot} g = -nmg$$

$$-kx - nmg = 0$$

Rearrange the equation(s) to isolate the unknown(s):

$$x = \frac{-nmg}{k}$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$x = \frac{-(49)(2.0 \times 10^{-3} \text{ kg})(9.81 \text{ m/s}^2)}{(24 \text{ N/m})}$$

$$x = -4.0 \times 10^{-2} \text{ m}$$

4. EVALUATE

Forty-nine shrews of 2.0 g each provide a total mass of about 0.1 kg, or a weight of just under I N. From the value of the spring constant, a force of 1 N should displace the spring by 1/24 of a meter, or about 4 cm. This indicates that the final result is consistent with the rest of the data.

ADDITIONAL PRACTICE

1. The largest meteorite of lunar origin reportedly has a mass of 19 g. If the meteorite placed on a scale whose spring constant is 83 N/m, what is the compression of the spring?

- 2. In 1952, a great rainfall hit the island of Reunion in the Indian Ocean. In less than 24 h, 187 kg of rain fell on each square meter of soil. If a 187 kg mass is placed on a scale that has a spring constant of 1.53 × 10⁴ N/m, how far is the spring compressed?
- 3. The largest tigers, and therefore the largest members of the cat family, are the Siberian tigers. Male Siberian tigers are reported to have an average mass of about 389 kg. By contrast, a variety of very small cat that is native to India has an average adult mass of only 1.5 kg. Suppose this small cat is placed on a spring scale, causing the spring to be extended from its equilibrium position by 1.2 mm. How far would the spring be extended if a typical male Siberian tiger were placed on the same scale?
- 4. The largest known crab is a giant spider crab that had a mass of 18.6 kg. The distance from the end of one of this crab's claws to the end of the other claw measured about 3.7 m. If this particular giant spider crab were hung from an elastic band so that the elongation of the band was equal to the crab's claw span, what would be the spring constant of the elastic band?
- 5. The CN Tower in Toronto, Canada, is 533 m tall, making it the world's tallest free-standing structure. Suppose an unusually long bungee cord is attached to the top of the CN Tower. The equilibrium length of the cord is equal to one-third the height of the tower. When a test mass of 70.0 kg is attached, the cord stretches to a length that equals two-thirds of the tower's height. From this information, determine the spring constant of the bungee cord.
- **6.** The largest ruby in the world may be found in New York. This ruby is 109 mm long, 91 mm wide, and 58 mm thick, making its volume about 575 cm³. (By comparison, the world's largest diamond, the Star of Africa, has a volume of just over 30 cm³.)
 - **a.** If the ruby is attached to a vertically hanging spring with a spring constant of 2.00×10^2 N/m so that the spring is stretched 15.8 cm what is the gravitational force pulling the spring?
 - b. What is the mass of the jewel?
- **7.** Mauna Kea on the island of Hawaii stands 4200 m above sea level. However, when measured from the island's sea-submerged base, Mauna Kea has a height of 10 200 m, making it the tallest single mountain in the world. If you have a 4.20×10^3 m elastic cord with a spring constant of 3.20×10^{-2} N/m, what force would stretch the spring to 1.02×10^4 m?
- **8.** Rising 348 m above the ground, La Gran Piedra in Cuba is the tallest rock on Earth. Suppose an elastic band 2.00×10^2 m long hangs vertically off the top of La Gran Piedra. If the band's spring constant is 25.0 N/m, how large must a mass be if, when it is attached to the band, it causes the band to stretch all the way to the ground?

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Problem 11 B

PERIOD OF A SIMPLE PENDULUM

PROBLEM

Two friends in France use a pendulum hanging from the world's highest railroad bridge to exchange messages across a river. One friend attaches a letter to the end of the pendulum and releases it so that the pendulum swings across the river to the other friend. The bridge is 130.0 m above the river. How much time is needed for the letter to make one swing across the river? Assume the river is 16.0 m wide.

SOLUTION

Given:

$$L = 130.0 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

Unknown:

t = time required for pendulum to cross river = T/2 = ?

Use the equation for the period of a simple pendulum. Then divide the period by two to find the time of one swing across the river. The width of the river is not needed to calculate the answer, but it must be small compared to the length of the pendulum in order to use the equations for simple harmonic motion.

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{130.0 \text{ m}}{9.81 \text{ m/s}^2}} = 22.9 \text{ s}$$

$$t = \frac{T}{2} = \frac{22.9 \text{ s}}{2} = \boxed{11.4 \text{ s}}$$

ADDITIONAL PRACTICE

- 1. An earthworm found in Africa was 6.7 m long. If this worm were a simple pendulum, what would its period be?
- 2. The shortest venomous snake, the spotted dwarf adder, has an average length of 20.0 cm. Suppose this snake hangs by its tail from a branch and holds a heavy prey with its jaws, simulating a pendulum with a length of 15.0 cm. How long will it take the snake to swing through one period?
- **3.** If bamboo, which can grow 88 cm in a day, is grown for four days and then used to make a simple pendulum, what will be the pendulum's period?
- **4.** A simple pendulum with a frequency of 6.4×10^{-2} Hz is as long as the largest known specimen of Pacific giant seaweed. What is this length?
- **5.** The deepest permafrost is found in Siberia, Russia. Suppose a shaft is drilled to the bottom of the frozen layer, and a simple pendulum with a length equal to the depth of the shaft oscillates within the shaft. In 1.00 h the pendulum makes 48 oscillations. Find the depth of the permafrost.
- **6.** Ganymede, the largest of Jupiter's moons, is also the largest satellite in the solar system. Find the acceleration of gravity on Ganymede if a simple pendulum with a length of 1.00 m has a period of 10.5 s.

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Problem 11 C

PERIOD OF A MASS-SPRING SYSTEM

PROBLEM

A large pearl was found in the Philippines in 1934. Suppose the pearl is placed on a spring scale whose spring constant is 362 N/m. If the scale's platform oscillates with a frequency of 1.20 Hz, what is the mass of the pearl?

SOLUTION

Given:

k = 362 N/m

f = 1.20 Hz

Unknown:

m = 3

Use the equation for the period of a mass-spring system. Then express the period in terms of frequency (T = 1/f).

$$T = 2\pi \sqrt{\frac{m}{k}} = \frac{1}{f}$$

$$m = \frac{k}{4\pi^2 f^2} = \frac{362 \text{ N/m}}{4\pi^2 (1.20 \text{ Hz})^2} = \boxed{6.37 \text{ kg}}$$

ADDITIONAL PRACTICE

- **1.** The hummingbird makes a humming sound with its wings, which beat with a frequency of 90.0 Hz. Suppose a mass is attached to a spring with a spring constant of 2.50×10^2 N/m. How large is the mass if its oscillation frequency is 3.00×10^{-2} times that of a hummingbird's wings?
- **2.** In 1986, a 35×10^3 kg watch was demonstrated in Canada. Suppose this watch is placed on a huge trailer that rests on a lightweight platform, and that oscillations equal to 0.71 Hz are induced. Find the trailer's mass if the platform acts like a spring scale with a spring constant equal to 1.0×10^6 N/m.
- **3.** A double coconut can grow for 10 years and have a mass of 20.0 kg. If a 20.0 kg double coconut oscillates on a spring 42.7 times each minute, what is the spring constant of the spring?
- **4.** The monument commemorating the Battle of San Jacinto in Texas stands almost 2.00×10^2 m and is topped by a 2.00×10^5 kg star. Imagine that a 2.00×10^5 kg mass is placed on a spring platform. The platform requires 0.80 s to oscillate from the top to the bottom positions. What is the spring constant of the spring supporting the platform?
- **5.** Suppose a 2662 kg giant seal is placed on a scale and produces a 20.0 cm compression. If the seal and spring system are set into simple harmonic motion, what is the period of the oscillations?
- **6.** On average, a newborn human's mass is just over 3.0 kg. However, in 1955, a 10.2 kg boy was born in Italy. Suppose this baby is placed in a crib hanging from springs with a total spring constant of 2.60×10^2 N/m. If the cradle is rocked with simple harmonic motion, what is its period?

Problem \|D

WAVE SPEED

PROBLEM

The world's largest guitar, which was built by high school students in Indiana, has strings that are 9.0 m long. The fundamental vibration that can be induced on each string has a wavelength equal to twice the string's length. If the wave speed in a string is 9.0×10^2 m/s, what is the frequency of vibration?

SOLUTION

Given:

 $f = 50.0 \, \text{Hz}$

L = 9.0 m

Unknown:

 $\nu = ?$

Use the equation for the speed of a wave. The wavelength is equal to twice the length of the string ($\lambda = 2L$).

$$v = f\lambda = f(2L) = (50.0 \text{ Hz})[(2)(9.0 \text{ m})] = 9.0 \times 10^2 \text{ m/s}$$

ADDITIONAL PRACTICE

- (1. The speed of sound in sea water is about 1530 m/s. If a sound wave has a frequency of 2.50×10^2 Hz, what is its wavelength in sea water?
- 2. Cicadas produce a sound that has a frequency of 123 Hz. What is the wavelength of this sound in the air? The speed of sound in air is 334 m/s.
- **3.** Human fingers are very sensitive, detecting vibrations with amplitudes as low as 2.0×10^{-5} m. Consider a sound wave with a wavelength exactly 1000 times greater than the lowest amplitude detectable by fingers. What is this wave's frequency?
- **4.** A nineteenth-century fisherman's cottage in England is only 2.54 m long. Suppose a fisherman whistles inside the cottage, producing a note that has a wavelength that exactly matches the length of the house. What is the whistle's frequency? The speed of sound in air is 334 m/s.
- **5.** The lowest vocal note in the classical repertoire is low D (f = 73.4 Hz), which occurs in an aria in Mozart's opera *Die Entführung aus dem Serail.* If low D has a wavelength of 4.50 m, what is the speed of sound in air?
- **6.** The highest-pitched sound that a human ear can detect is about 21 kHz. On the other hand, dolphins can hear ultrasound with frequencies up to 280 kHz. What is the speed of sound in water if the wavelength of ultrasound with a frequency of 2.80×10^5 Hz is 0.510 cm? How long would it take this sound wave to travel to a dolphin 3.00 km away?