

## Holt Physics

**Problem 3A -2****FINDING RESULTANT MAGNITUDE AND DIRECTION****PROBLEM**

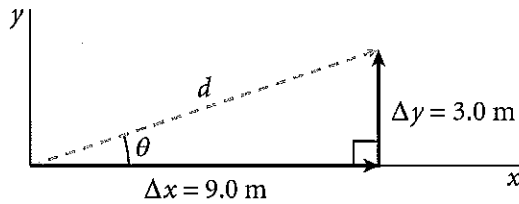
A hummingbird flies 9.0 m horizontally and then flies up for 3.0 m. What is the bird's resultant displacement?

**SOLUTION**

**1. DEFINE** Given:  $\Delta x = 9.0 \text{ m}$      $\Delta y = 3.0 \text{ m}$

Unknown:  $d = ?$      $\theta = ?$

Diagram:



**2. PLAN** Choose the equation(s) or situation: The Pythagorean theorem can be used to find the magnitude of the hummingbird's displacement. The direction of the displacement can be found using the tangent function.

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\tan \theta = \frac{\Delta y}{\Delta x}$$

Rearrange the equation(s) to isolate the unknown(s):

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\theta = \tan^{-1} \left( \frac{\Delta y}{\Delta x} \right)$$

**3. CALCULATE** Substitute the values into the equation(s) and solve:

$$d = \sqrt{(9.0 \text{ m})^2 + (3.0 \text{ m})^2} = \sqrt{81 \text{ m}^2 + 9.0 \text{ m}^2} = \sqrt{9.0 \times 10^1 \text{ m}^2}$$

$$d = \boxed{9.5 \text{ m}}$$

$$\theta = \tan^{-1} \left( \frac{3.0 \text{ m}}{9.0 \text{ m}} \right)$$

$$\theta = \boxed{18^\circ \text{ above horizontal}}$$

**4. EVALUATE** The resultant displacement ( $d$ ) is only slightly larger than the largest component ( $\Delta x$ ), as is the case for small angles ( $\theta \approx 20^\circ$ ).

**ADDITIONAL PRACTICE**

- A tiger paces back and forth along the front of its cage, which is 8 m wide. The tiger starts from the right side of the cage, paces to the left side, then back to the right side, and finally back to the left.
  - What total distance has the tiger paced?
  - What is the tiger's resultant displacement?
- A particular type of rubber ball is able to bounce to 0.90 times the height from which it is dropped. The ball is dropped from a height of 0.91 m, but it falls slightly away from the vertical, so that by the time it has bounced to its new height it has undergone a horizontal displacement of 0.11 m. What is the ball's resultant displacement from its initial height to its maximum height after one bounce?
- A helicopter flies 165 m horizontally and then moves downward to land 45 m below. What is the helicopter's resultant displacement?
- A toy parachute is dropped from an open window that is 13.0 m above the ground. If the parachute travels 9.0 m horizontally, what is the resultant displacement?
- An octopus swims 36.0 m east, 42.0 m north, and then rises 17.0 m toward the surface of the water. What is the octopus's displacement?  
(TWO-DIMENSIONAL METHOD: Visualize a horizontal and a vertical triangle. Find the horizontal resultant; use that with the vertical distance to calculate the final resultant. Studying this method can lead to understanding the easier three-dimensional solution in the solutions manual.)
- An airplane taxis to the end of a runway before taking off. The magnitude of the plane's total displacement is 599 m. If the northern component of this displacement is 89 m, what is the displacement's eastern component? What is the direction of the total displacement?
- The straightest stretch of railroad tracks in the world extends for 478 km in southwestern Australia. A train traveling along these tracks is displaced to the south by about 42 km. What is the train's displacement to the west? What is the direction of the total displacement?
- Before the widespread use of steamships, sailing from Europe to North America was accomplished by use of the "trade winds." The trade winds move from the northeast to the southwest between  $30^\circ$  and  $60^\circ$  latitude in the northern hemisphere. A ship sailing from Europe to the Caribbean Sea would first travel southward to the Canary Islands, off the coast of North Africa, and then use the trade winds to sail west. Suppose a ship travels south from Iceland to the Canary Islands, and then west to Florida. The ship's total displacement is 7400 km at  $26^\circ$  south of west. If the ship sails 3200 km south from Iceland to the Canary Islands, how large is the western component of its journey?

- 9.** The Palm Springs Aerial Tramway extends 3.88 km from the Valley Station, which is located 0.8 km above sea level, to the Mountain Station atop San Jacinto Peak. The actual path of the tramway's cables is not along a straight line, but if it were, the horizontal displacement of the tramway would be 3.45 km. How far is San Jacinto Peak above sea level?
- 10.** The islands that form the Tristan da Cunha Group in the South Atlantic Ocean are considered to be the most remote places in the world: the next nearest inhabited island is 2400 km away. If you sail from Capetown, South Africa, in a south by southwest direction, you must travel  $2.9 \times 10^3$  km before reaching the Tristan da Cunha islands. If the western component of your displacement is  $2.8 \times 10^3$  km, what is your displacement south? In what direction is the resultant displacement?

## Holt Physics

**Problem 3B -2****RESOLVING VECTORS****PROBLEM**

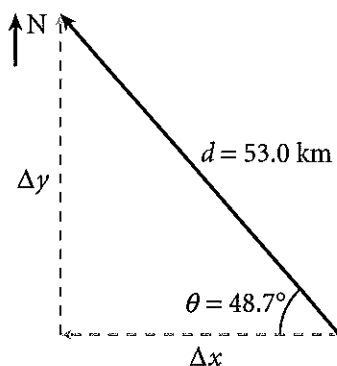
The straight stretch of Interstate Highway 5 from Mettler, California, to a point near Buttonwillow, California, is 53.0 km long and makes an angle of  $48.7^\circ$  north of west. What are the northern and western components of this highway segment?

**SOLUTION**

**1. DEFINE** Given:  $d = 53.0$  km  $\theta = 48.7^\circ$  north of west

Unknown:  $\Delta x = ?$   $\Delta y = ?$

Diagram:



**2. PLAN** Choose the equation(s) or situation: Because the axes are perpendicular, the sine and cosine functions can be used to find the components.

$$\sin \theta = \frac{\Delta y}{d}$$

$$\cos \theta = \frac{\Delta x}{d}$$

Rearrange the equation(s) to isolate the unknown(s):

$$\Delta x = d (\cos \theta)$$

$$\Delta y = d (\sin \theta)$$

**3. CALCULATE** Substitute the values into the equation(s) and solve:

$$\Delta x = (53.0 \text{ km})(\cos 48.7^\circ)$$

$$\Delta x = \boxed{35.0 \text{ km, west}}$$

$$\Delta y = (53.0 \text{ km})(\sin 48.7^\circ)$$

$$\Delta y = \boxed{39.8 \text{ km, north}}$$

**4. EVALUATE** Using the Pythagorean theorem to check the answers confirms the magnitudes of the components.

$$d^2 = \Delta x^2 + \Delta y^2$$

$$(53.0 \text{ km})^2 = (35.0 \text{ km})^2 + (39.8 \text{ km})^2$$

$$2.80 \times 10^3 \text{ km}^2 = 1.22 \times 10^3 \text{ km}^2 + 1.58 \times 10^3 \text{ km}^2$$

$$2.80 \times 10^3 \text{ km}^2 = 2.80 \times 10^3 \text{ km}^2$$

**ADDITIONAL PRACTICE**

1. The distance from an observer on the plain to the top of a nearby mountain is 5.3 km, and the angle between this line and the horizontal is  $8.4^\circ$ . How tall is the mountain?
2. A bowling ball is released at the near right corner of a bowling lane and travels 19.1 m at an angle of  $3.0^\circ$  with respect to the lane's length. The ball reaches the far left corner of the lane, where it knocks over the "7" pin. What is the width of the lane?
3. A skyrocket travels 113 m at an angle of  $82.4^\circ$  with respect to the ground and toward the south. What is the rocket's horizontal displacement?
4. A hot-air balloon descends with a velocity of 55 km/h at an angle of  $37^\circ$  below the horizontal. What is the vertical velocity of the balloon?
5. A billiard ball travels 2.7 m at an angle of  $13^\circ$  with respect to the long side of the table. What are the components of the ball's displacement?
6. One hole at a certain miniature golf course extends for about 60 m. A golf ball on this hole travels with a velocity of 1.20 m/s at  $14.0^\circ$  east of north. What are the eastern and northern components of the ball's velocity?
7. The Very Large Array in western New Mexico consists of several radio telescopes that can be rearranged along railroad tracks. The largest of these arrangements has the telescopes positioned in a "Y" pattern for 18 km along three separate tracks. Suppose an electrician inspects the instruments in each antenna from the end of the northern track to the end of the southwestern track. If the electrician's resultant displacement is 31.2 km at  $30.0^\circ$  west of south, what are the southern and western components of the displacement?
8. Barnard's Star is the closest star to Earth after the sun and the triple star Alpha Centauri. Barnard's Star has a velocity of 165.2 km/s at an angle of  $32.7^\circ$  away from its forward motion. What are the forward and side components of this velocity?
9. A tiger leaps with an initial velocity of 55.0 km/h at an angle of  $13.0^\circ$  with respect to the horizontal. What are the components of the tiger's velocity?
10. A certain type of balloon is designed to ascend rapidly. Suppose this balloon has a velocity 13.9 m/s at  $26.0^\circ$  above the horizontal and  $24.0^\circ$  east of north. What are the upward, northern, and eastern components of the balloon's velocity? (HINT: Draw horizontal and vertical right triangles whose sides represent the velocity's components.)

Holt Physics

# Problem 3C -2

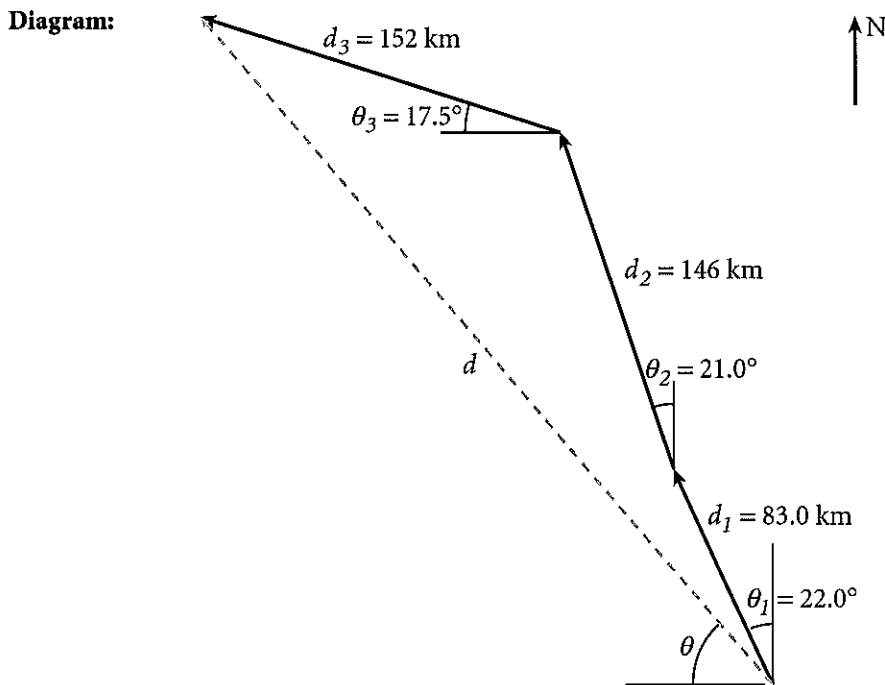
**ADDING VECTORS ALGEBRAICALLY**

**PROBLEM**

The southernmost point in the United States is called South Point, and is located at the southern tip of the large island of Hawaii. A plane designed to take off and land in water leaves South Point and flies to Honolulu, on the island of Oahu, in three separate stages. The plane first flies 83.0 km at 22.0° west of north from South Point to Kailua Kona, Hawaii. The plane then flies 146 km at 21.0° west of north from Kailua Kona to Kahului, on the island of Maui. Finally, the plane flies 152 km at 17.5° north of west from Kahului to Honolulu. What is the plane's resultant displacement?

**SOLUTION**

- 1. DEFINE**
- |                 |                         |                                       |
|-----------------|-------------------------|---------------------------------------|
| <b>Given:</b>   | $d_1 = 83.0 \text{ km}$ | $\theta_1 = 22.0^\circ$ west of north |
|                 | $d_2 = 146 \text{ km}$  | $\theta_2 = 21.0^\circ$ west of north |
|                 | $d_3 = 152 \text{ km}$  | $\theta_3 = 17.5^\circ$ north of west |
| <b>Unknown:</b> | $d = ?$                 | $\theta = ?$                          |



- 2. PLAN** Choose the equation(s) or situation: Express the components of each vector in terms of sine or cosine functions.

$$\begin{aligned} \Delta x_1 &= d_1 (\sin \theta_1) & \Delta y_1 &= d_1 (\cos \theta_1) \\ \Delta x_2 &= d_2 (\sin \theta_2) & \Delta y_2 &= d_2 (\cos \theta_2) \\ \Delta x_3 &= d_3 (\cos \theta_3) & \Delta y_3 &= d_3 (\sin \theta_3) \end{aligned}$$

Note that the angles  $\theta_1$  and  $\theta_2$  are with respect to the  $y$  axis (north), and so the  $x$  components are in terms of  $\sin \theta$ . Write the equations for  $\Delta x_{tot}$  and  $\Delta y_{tot}$ , the components of the total displacement.

$$\begin{aligned}\Delta x_{tot} &= \Delta x_1 + \Delta x_2 + \Delta x_3 \\ &= d_1(\sin \theta_1) + d_2(\sin \theta_2) + d_3(\cos \theta_3) \\ \Delta y_{tot} &= \Delta y_1 + \Delta y_2 + \Delta y_3 \\ &= d_1(\cos \theta_1) + d_2(\cos \theta_2) + d_3(\sin \theta_3)\end{aligned}$$

Use the components of the total displacement, the Pythagorean theorem, and the tangent function to calculate the total displacement.

$$\begin{aligned}d &= \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} \\ \theta &= \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right)\end{aligned}$$

**3. CALCULATE**

**Substitute the values into the equation(s) and solve:**

$$\begin{aligned}\Delta x_{tot} &= (83.0 \text{ km})(\sin 22.0^\circ) + (146 \text{ km})(\sin 21.0^\circ) + (152 \text{ km}) \\ &\quad (\cos 17.5^\circ) \\ &= 31.1 \text{ km} + 52.3 \text{ km} + 145 \text{ km} \\ &= 228 \text{ km}\end{aligned}$$

$$\begin{aligned}\Delta y_{tot} &= (83.0 \text{ km})(\cos 22.0^\circ) + (146 \text{ km})(\cos 21.0^\circ) + (152 \text{ km}) \\ &\quad (\sin 17.5^\circ) \\ &= 259 \text{ km}\end{aligned}$$

$$\begin{aligned}d &= \sqrt{(228 \text{ km})^2 + (259 \text{ km})^2} = \\ &= \sqrt{5.20 \times 10^4 \text{ km}^2 + 6.71 \times 10^4 \text{ km}^2} = \sqrt{11.91 \times 10^4 \text{ km}^2} \\ &= \boxed{345.1 \text{ km}}\end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{259 \text{ km}}{228 \text{ km}}\right) = \boxed{48.6^\circ \text{ north of west}}$$

**4. EVALUATE**

If the diagram is drawn to scale, compare the calculated results to the drawing. The length of the drawn resultant is fairly close to the scaled magnitude for  $d$ , while the angle appears to be slightly greater than  $45^\circ$ .

**ADDITIONAL PRACTICE**

- U.S. Highway 212 extends 55 km at  $37^\circ$  north of east between Newell and Mud Butte, South Dakota. It then continues for 66 km nearly due east from Mud Butte to Faith, South Dakota. If you drive along this part of U.S. Highway 212, what will be your total displacement?
- Wrigley Field is one of only three original major-league baseball fields that are still in use today. Suppose you want to drive to Wrigley Field from the corner of 55th Street and Woodlawn Avenue, about 14 miles south of Wrigley Field. Although not the fastest or most direct route, the most straightforward way to reach Wrigley Field is to drive 4.1 km west on 55th Street to Halsted Street, then turn north and drive 17.3 km on Halsted until you reach Clark Street. Turning on Clark, you will reach Wrigley Field after traveling 1.2 km at an angle of  $24.6^\circ$  west of north. What is your resultant displacement?

3. A bullet traveling 850 m ricochets from a rock. The bullet travels another 640 m, but at an angle of  $36^\circ$  from its previous forward motion. What is the resultant displacement of the bullet?
4. The cable car system in San Francisco is the last of its kind that is still in use in the United States. It was originally designed to transport large numbers of people up the steep hills on which parts of the city are built. If you ride seven blocks on the Powell Street cable car from the terminal at Market Street to Pine Street, you will travel  $2.00 \times 10^2$  m on level ground, then  $3.00 \times 10^2$  m at an incline of  $3.0^\circ$  to the horizontal, and finally  $2.00 \times 10^2$  m at  $8.8^\circ$  to the horizontal. What will be your resultant displacement?
5. An Arctic tern flying to Antarctica encounters a storm. The tern changes direction to fly around the storm. If the tern flies 46 km at  $15^\circ$  south of east, 22 km at  $13^\circ$  east of south, and finally 14 km at  $14^\circ$  west of south, what is the tern's resultant displacement?
6. A technique used to change the direction of space probes, as well as to give them additional speed, is to use the gravitational pull of nearby planet. This technique was first used with the Voyager probes. Voyager 2 had traveled about  $6.3 \times 10^8$  km when it reached Jupiter. Jupiter's gravity changed Voyager's direction by  $68^\circ$ . The probe then traveled about  $9.4 \times 10^8$  km when it reached Saturn, and its direction was changed by  $94^\circ$ . Voyager 2 was now redirected; it encountered Uranus after traveling  $3.4 \times 10^9$  km from Saturn. Use this information to calculate the resultant displacement of Voyager 2 as it traveled from Earth to Uranus.
7. The city of Amsterdam, in the Netherlands, has several canals that connect different sections of the city. Suppose you take a barge trip to the harbor, starting at a point near the northwest corner of the Vondelpark. You would sail  $2.50 \times 10^3$  m at  $58.5^\circ$  north of east, 375 m at  $21.8^\circ$  north of east, and 875 m at  $21.5^\circ$  east of north. What would be your resultant displacement?
8. The elevated train, or "L," in Chicago is a major source for mass transit in that city. One of the lines extends from Jefferson Park, in the northwest part of town, to the Clark Street station downtown. The route of this line runs 5.0 km at  $36.9^\circ$  south of east, 1.5 km due south, 8.5 km at  $42.2^\circ$  south of east, and 0.8 km due east. What is the resultant displacement of an "L" train from Jefferson Park to Clark Street?
9. A billiard table is positioned with its long side parallel to north. A cue ball is then shot so that it travels 1.41 m at an angle of  $45.0^\circ$  west of north, is deflected by the table's left side, and continues to move 1.98 m east of north at an angle of  $45.0^\circ$ . The ball is then deflected by the table's right side, so that it moves 0.42 m west of north at an angle of  $45.0^\circ$ . After a reflection on the north end of the table, the ball travels 1.56 m at an angle of  $45.0^\circ$  south of west. Determine the resultant displacement of the cue ball.



- 10.** Hurricane Iniki was the most destructive cyclone to have crossed the Hawaiian Islands in the twentieth century. Its path was also unusual: it moved south of the islands for 790 km at an angle of  $18^\circ$  north of west, then moved due west for 150 km, turned north and continued for 470 km, and finally turned back  $15^\circ$  east of north and moved 240 km to cross the island of Kauai. What was the resultant displacement of Hurricane Iniki?

## Holt Physics

**Problem 3D -2****PROJECTILES LAUNCHED HORIZONTALLY****PROBLEM**

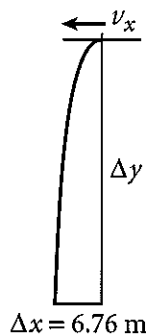
Although not the fastest or tallest or steepest roller coaster in the world, the “High Roller” roller coaster atop the Stratosphere Tower, in Las Vegas, Nevada, is the highest. Suppose that during construction of the ride a metal bolt was accidentally knocked horizontally off the edge of the Stratosphere. If the bolt’s initial speed was 0.80 m/s, it would have traveled 6.76 m in the horizontal direction before hitting the ground. Use this information to calculate how tall the Stratosphere Tower is.

**SOLUTION**

**1. DEFINE** Given:  $v_x = 0.80 \text{ m/s}$   
 $\Delta x = 6.76 \text{ m}$   
 $g = 9.81 \text{ m/s}^2$

Unknown:  $\Delta y = ?$

Diagram:



**2. PLAN** Choose the equation(s) or situation: The magnitude of the vertical displacement is given by the equation for falling bodies with no initial vertical velocity.

$$\Delta y = -\frac{1}{2} g \Delta t^2$$

The magnitude for horizontal displacement is given by the equation for displacement at constant velocity.

$$\Delta x = v_x \Delta t$$

Rearrange the equation(s) to isolate the unknown(s): Substitute for  $\Delta t$  in the falling-body equation.

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = -\frac{1}{2} g \left( \frac{\Delta x}{v_x} \right)^2$$

**3. CALCULATE** Substitute the values into the equation(s) and solve:

$$\Delta y = -\frac{1}{2} (9.81 \text{ m/s}^2) \left( \frac{6.76 \text{ m}}{0.80 \text{ m/s}} \right)^2$$

$$= 350 \text{ m}$$

height of building = 350 m

- 4. EVALUATE** The solution can be checked by using both equations to solve for  $\Delta t$ . From the equation for falling bodies,  $\Delta t$  is found to be 8.4 s. From the equation for horizontal displacement,  $\Delta t$  is 8.4 s. Both times are the same, so  $\Delta y$  is correctly calculated.

### ADDITIONAL PRACTICE

1. Lookout Mountain, which overlooks the Tennessee River Valley near Chattanooga, Tennessee, was of great strategic importance during the Civil War. Today, some of the artillery used in the war remain at the park located on top of the mountain. Suppose one of these cannons fired a projectile horizontally with a speed of 430 m/s, so that the projectile landed at a horizontal distance of 4020 m from the cannon. How high would the ridge of the mountain be with respect to the valley below?
2. In 1977, a helicopter at the heliport atop the 59-story Pan Am building in New York fell over, causing the rapidly-turning rotor blades to splinter. One of these fragments landed about 101 m away, near the corner of Madison Avenue and 43rd Street. Suppose the fragment moved off the building horizontally with a speed of 14.25 m/s. Use this information to find the height of the Pan Am building.
3. The LZ N07 is a newly designed airship in the manner of the old Zeppelin airships built in Germany between 1908 and 1940. New technology has made the LZ N07 more efficient and safe, as well as speedier. This airship can travel with a horizontal speed of up to  $1.30 \times 10^2$  km/h. If a parcel is dropped from this airship, so that it lands 135 m in front of the spot over which it was released, how far above the ground is the airship?
4. The shape of Sugarloaf mountain, in Rio de Janeiro, Brazil, is such that, if you were to kick a soccer ball hard enough, it could land near the base of the mountain without hitting the mountain's side. Suppose the ball is kicked horizontally with an initial speed of 9.37 m/s. If the ball travels a horizontal distance of 85.0 m, how tall is the mountain?
5. Although many structures taller than 500 m have been designed, few have been built due to practical limitations, such as cost and safety. In light of this, the Bionic Tower in Hong Kong may never be more than a design. If it is built, the Bionic Tower will provide working space for 100,000 people, and transport them using over 300 elevators. Suppose a plate-glass window falls out of place from the top floor of the Bionic Tower. Although the window's speed is only 6.32 cm/s in the horizontal direction, the window will still have a horizontal displacement of 1.00 m once it hits the street below. Use this information to calculate the proposed height of the Bionic Tower.
6. A squirrel on a limb near the top of a tree loses its grip on a nut, so that the nut slips away horizontally at a speed of 10.0 cm/s. If the nut lands at a horizontal distance of 18.6 cm, how high above the ground is the squirrel?

7. A lunch pail is accidentally kicked off a steel beam on a building under construction. Suppose the initial horizontal speed is 1.50 m/s. How far does the lunch pail fall after it travels 3.50 m horizontally?
8. If the building in problem 7 is  $2.50 \times 10^2$  m tall, and the lunch pail is knocked off the top floor, what will be the horizontal displacement of the lunch pail when it reaches the ground?
9. What is the velocity of the lunch pail in problem 8 when it reaches the ground?
10. What is the range of an arrow shot horizontally at 85.3 m/s if it is initially 1.50 m above the ground?

## Holt Physics

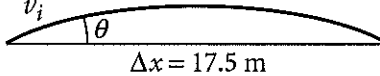
**Problem 3E -2****PROJECTILES LAUNCHED AT AN ANGLE****PROBLEM**

A flying fish leaps out of the water with a speed of 15.3 m/s. Normally these fish use winglike fins to glide about 40 m before reentering the ocean, but in this case the fish fails to use its “wings” and so only travels horizontally about 17.5 m. At what angle with respect to the water’s surface does the fish leave the water? Use the trigonometric identity  $2(\sin\theta)(\cos\theta) = \sin(2\theta)$  to solve for  $\theta$ .

**SOLUTION**

**1. DEFINE** **Given:**  $v_i = 15.3 \text{ m/s}$   
 $\Delta x = 17.5 \text{ m}$   
 $g = 9.81 \text{ m/s}^2$

**Unknown:**  $\theta = ?$

**Diagram:** 

**2. PLAN** **Choose the equation(s) or situation:** The horizontal component of the fish’s initial velocity,  $v_x$ , is equal to the horizontal displacement divided by the time of the jump.

$$v_x = v_i (\cos \theta) = \frac{\Delta x}{\Delta t} \quad \Delta x = v_i \cos \theta \Delta t$$

The vertical displacement of the fish is given by the equation for falling bodies, with the vertical component of the initial velocity,  $v_y$ , used.

$$\Delta y = v_y \Delta t - \frac{1}{2} g \Delta t^2$$

Because the fish lands at the same vertical position from which it started,  $\Delta y = 0$ .

$$\Delta y = 0$$

$$v_y = v_i (\sin \theta) = \frac{1}{2} g \Delta t$$

**Rearrange the equation(s) to isolate the unknowns:** Substitute for  $\Delta t$  using the equation for horizontal velocity.

$$\Delta t = \frac{\Delta x}{v_i (\cos \theta)}$$

$$v_i (\sin \theta) = \frac{1}{2} g \left[ \frac{\Delta x}{v_i (\cos \theta)} \right]$$

$$(\sin \theta)(\cos \theta) = \frac{g \Delta x}{2 v_i^2}$$

Using the trigonometric identity allows a solution for  $\theta$  to be found.

$$(\sin \theta)(\cos \theta) = \frac{1}{2} [\sin(2\theta)]$$

$$\sin(2\theta) = \frac{g \Delta x}{v_i^2}$$

$$\theta = \frac{\sin^{-1}\left(\frac{g\Delta x}{v_i^2}\right)}{2}$$

**3. CALCULATE**

$$\theta = \frac{\sin^{-1}\left[\frac{(9.81 \text{ m/s}^2)(17.5 \text{ m})}{(15.3 \text{ m/s})^2}\right]}{2}$$

$$= \boxed{23.6^\circ \text{ above the horizontal}}$$

**4. EVALUATE**

Substituting the value for  $\theta$  into the original equations and solving for  $\Delta t$  produces a time of 1.25 s for both, thus confirming the result for  $\theta$ .

### ADDITIONAL PRACTICE

1. A baseball is thrown with an initial speed of 15.0 m/s. If the ball's horizontal displacement is 17.6 m, at what angle with respect to the ground is the ball pitched? Use the trigonometric identity  $2(\sin \theta)(\cos \theta) = \sin(2\theta)$  to solve for  $\theta$ .
2. A football is kicked so that its initial speed is 23.1 m/s. If the football reaches a maximum height of 16.9 m, at what angle with respect to the ground is the ball kicked?
3. Jackie Joyner-Kersey's record long jump is 7.49 m. Suppose she ran 9.50 m/s to jump this horizontal distance. At what angle above the horizontal did she jump? Use the trigonometric identity  $2(\sin \theta)(\cos \theta) = \sin(2\theta)$  to solve for  $\theta$ .
4. The small jumping spiders make up for their size by their ability to leap relatively large distances. Some can jump fifty times the length of their bodies. Suppose a jumping spider leaps a horizontal distance of 18.5 cm with an initial speed of about 141 cm/s. At what angle above the horizontal would a spider with this speed have to leap in order to travel a range of 18.5 cm? Use the trigonometric identity  $2(\sin \theta)(\cos \theta) = \sin(2\theta)$  to solve for  $\theta$ .
5. Olympic platform divers jump from a diving board that is 10.0 m above the water. Suppose a diver jumps from the board with an initial speed of 6.03 m/s. The diver reaches a maximum height of 11.7 m above the water, and lands in the water at a horizontal distance of 3.62 m from the end of the board. At what angle with respect to the board does the diver leave the board?
6. A ball is thrown from a roof with a speed of 10.0 m/s and an angle of  $37.0^\circ$  with respect to the horizontal. What are the vertical and horizontal components of the ball's displacement 2.5 s after it is thrown?
7. A downed pilot fires a flare from a flare gun. The flare has an initial speed of 250 m/s and is fired at an angle of  $35^\circ$  to the ground. How long does it take for the flare to reach its maximum altitude?

- 8.** In the sport of ski jumping, a skier travels down the slope of a hill until he or she reaches the takeoff. The takeoff is slanted slightly below the horizontal, so that the skier is able to travel in the air just above the ground. Suppose a skier leaves the takeoff and lands 73.0 m horizontally beyond the takeoff and  $-52.8$  m below the takeoff. If the takeoff angle is  $-8.00^\circ$  below the horizontal, what is the skier's initial speed?
- 9.** A shingle slides down a roof having a  $30.0^\circ$  pitch and falls off with a speed of 2.0 m/s. How long will it take to hit the ground 45 m below?
- 10.** A hole at a miniature golf course requires the ball to roll up a ramp, fly over a small stream, and then land on the green beyond the stream. The stream is 0.46 m wide, and the cup is 4.00 m beyond the stream's edge. The ramp makes an angle of  $41.0^\circ$  with the horizontal, and its upper edge is 0.35 m above the green. What must the ball's initial speed be in order for the ball to fly over the water and land directly in the cup?

## Holt Physics

**Problem 3F -2****RELATIVE VELOCITY****PROBLEM**

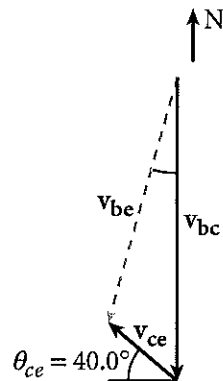
A polar bear swims 2.60 m/s south relative to the water. The bear is swimming against a current that moves 0.78 m/s at an angle of  $40.0^\circ$  north of west, relative to Earth. How long will it take the polar bear to reach the shore, which is 5.50 km to the south?

**SOLUTION****1. DEFINE**

**Given:**  $v_{bc} = 2.60$  m/s due south (velocity of the *bear*, *b*, with respect to the *current*, *c*)  
 $v_{ce} = 0.78$  m/s at  $40.0^\circ$  north of west (velocity of the *current*, *c*, with respect to *Earth*, *e*)  
 $\Delta y = 5.50$  km, south

**Unknown:**  $\Delta t = ?$

**Diagram:**



- 2. PLAN** **Choose the equation(s) or situation:** To find  $v_{be}$ , write the equation so that the subscripts of the vectors on the right begin with *b* and end with *e*.

$$\mathbf{v}_{be} = \mathbf{v}_{bc} + \mathbf{v}_{ce}$$

Because vectors  $\mathbf{v}_{bc}$  and  $\mathbf{v}_{ce}$  are not perpendicular, their *x* and *y* components must be calculated. Aligning the positive *y* axis with north and treating west as the positive *x* direction for convenience, the following equations apply for the magnitude of the components of  $\mathbf{v}_{be}$ .

$$v_{x,be} = v_{x,bc} + v_{x,ce} = v_{x,ce} = v_{ce} (\cos \theta_{ce})$$

$$v_{y,be} = v_{y,bc} + v_{y,ce} = -v_{bc} + v_{ce} (\sin \theta_{ce})$$

From these components the magnitude and direction of  $\mathbf{v}_{be}$  could be found from the Pythagorean theorem and the tangent function, respectively. However, only the component  $v_{y,be}$  is needed to calculate the time required for the bear to swim in the negative *y* direction.

$$\Delta t = \frac{-\Delta y}{v_{y,be}}$$



Rearrange the equation(s) to isolate the unknown(s):

$$\Delta t = \frac{-\Delta y}{v_{y,be}} = \frac{-\Delta y}{[-v_{bc} + v_{ce}(\sin \theta_{ce})]}$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\begin{aligned}\Delta t &= \frac{-5.50 \text{ km}}{(-2.60 \text{ m/s} + 0.78 \text{ m/s})(\sin 40.0^\circ)} \\ &= \frac{-5.50 \text{ km}}{(-2.60 \text{ m/s} + 0.50 \text{ m/s})} = \frac{-5.50 \times 10^3 \text{ m}}{-2.10 \text{ m/s}}\end{aligned}$$

$$\Delta t = \boxed{2.62 \times 10^3 \text{ s, or 43 min 40 s}}$$

4. EVALUATE

Without the current, the polar bear would arrive about 500 s or 8.3 min sooner. The 500 s delay is about one fourth (25%) of the bear's swimming time without the current. This proportion is equal to the ratio of the current's northern component to the bear's velocity to the south.

### ADDITIONAL PRACTICE

1. A bird flies directly into a wind. If the bird's forward speed relative to the wind is 58.0 km/h and the wind's speed in the opposite direction is 55.0 km/h, relative to Earth, how long will it take the bird to fly 1.4 km?
2. A moving walkway at an airport has a velocity of 1.50 m/s to the west. A man rushing to catch his flight runs down the walkway with a velocity of 4.20 m/s to the west relative to the walkway. If the walkway is  $8.50 \times 10^2$  m long, how much time does the man save by running on the walkway as opposed to running on a non-moving surface?
3. The greatest average speed for a race car in the Daytona 500 is 286 km/h, which was achieved in 1980. Suppose a race car moving at this speed is in second place, being 0.750 km behind a car that is moving at a speed of 252 km/h. How long will it take the second-place car to catch up to the first-place car?
4. A mosquito can fly with a speed of 1.10 m/s with respect to the air. Suppose a mosquito flies east at this speed across a swamp. The mosquito is flying into a breeze that has a velocity of 5.0 km/h with respect to Earth and moves  $35^\circ$  west of south. If the swamp is 540 m across, how long will it take the mosquito to cross the swamp?
5. A glider descends with a velocity relative to the air of 150 km/h at an angle of  $7.0^\circ$  below the horizontal. Suppose that the glider encounters an updraft with a velocity relative to Earth of 15 km/h upward. How long will it take the glider to reach the ground if it encounters the updraft at 166 m? How long would it take for the glider to land without the updraft?
6. A flare gun is mounted on an automobile and fired perpendicular to the car's motion. The car's velocity with respect to Earth is 145 km/h to the north. The flare's velocity with respect to the car is 87 km/h to the west. What are the components of the flare's displacement with respect to Earth 0.45 s after the flare is launched?

7. An airship moving north at 55.0 km/h with respect to the air encounters a wind from  $17.0^\circ$  north of west. If the wind's speed with respect to Earth is 40.0 km/h, what is the airship's velocity with respect to Earth?
8. How far to the north and west does the airship in problem 7 travel after 15.0 minutes?
9. A torpedo fired at an anchored target moves against a current. Suppose the torpedo's velocity with respect to the current is 51 km/h east, and the current's velocity with respect to the target is 4.0 km/h south. If the torpedo hits the target in 14 s, how far away is the target from the point where the torpedo is launched? How far north of the target must the torpedo be launched in order to hit the target?
10. A sailboat travels south with a speed of 12.0 km/h with respect to the water. Suppose the boat encounters a current that has a velocity with respect to Earth of 4.0 km/h at  $15.0^\circ$  south of east. What is the sailboat's resultant velocity with respect to Earth?