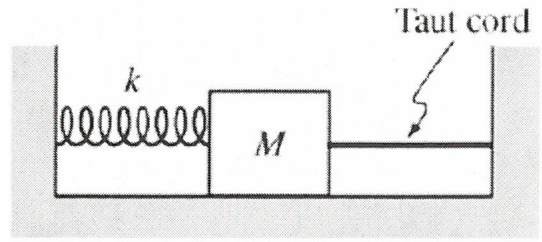
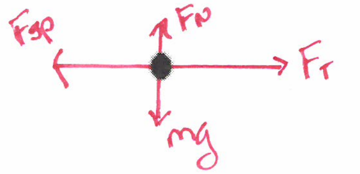
Pre- Exam – Unit 6 – FRQ – KEY

1) One end of a spring of spring constant k is attached to a wall, and the other end is attached to a block of mass M, as shown above. The block is pulled to the right, stretching the spring from its equilibrium position, and is then held in place by a taut cord, the other end of which is attached to the opposite wall. The spring and the cord have negligible mass, and the tension in the cord is F_T . Friction between the block and the surface is negligible. Express all algebraic answers in terms of M, k, F_T , and fundamental constants.



a) On the dot below that represents the block, draw and label a free-body diagram for the block.



b) Calculate the distance that the spring has been stretched from its equilibrium position.

$\Sigma F_{net} = ma$

$F_T - F_{sp} = m\cancel{a}^{\,0}$

$F_{sp} = \Delta x K$

$\Delta x K = F_T$

$\boxed{\Delta x = \frac{F_T}{K}}$

The cord suddenly breaks so that the block initially moves to the left and then oscillates back and forth.

c) Calculate the speed of the block when it has moved half the distance from its release point to its equilibrium position

E ? $V_0 = 0$

KE Usp @ $\frac{x}{2}$ KE Usp x

$U_{sp} + KE = U_{sp}$

$\frac{1}{2}K\left(\frac{x}{2}\right)^2 + \frac{1}{2}mV_m^2 = \frac{1}{2}Kx^2$

$Kx^2 - \frac{1}{4}Kx^2 = mV_m^2$

$\frac{3}{4}\frac{Kx^2}{m} = V_m^2$

$V_m^2 = \frac{3}{4}\frac{Kx^2}{m}$

$x = \frac{F_T}{K}$

$V_m^2 = \frac{3}{4}\frac{\cancel{K}}{m}\left(\frac{F_T^2}{K^2}\right)$

$V_m = \sqrt{\frac{3}{4}\frac{F_T^2}{mK}}$

$\boxed{V_m = \frac{F_T}{2}\sqrt{\frac{3}{mK}}}$

d) Calculate the time after the cord breaks until the block first reaches its position furthest to the left

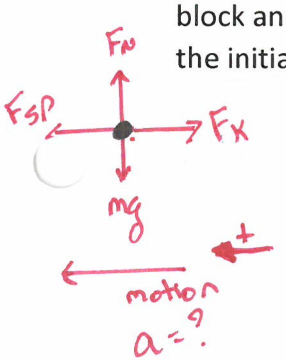
$T = 2\pi\sqrt{\frac{m}{K}}$

To Reach the position from the far left will take ½ of a period of oscillation

$T = \frac{1}{2}\cancel{2}\pi\sqrt{\frac{M}{K}}$

$\boxed{T = \pi\sqrt{\frac{M}{K}}}$

e) Suppose instead that friction is not negligible and that the coefficient of kinetic friction between the block and the surface is μ_k . After the cord breaks, the block again initially moves to the left. Calculate the initial acceleration of the block just after the cord breaks.



$F_{net} = ma$

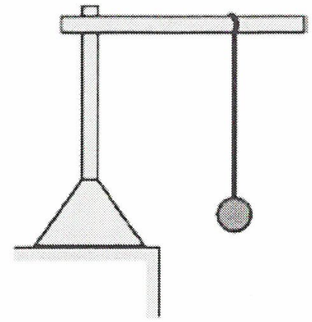
$F_{sp} - F_k = ma$

$F_k = \mu_k F_N = \mu_k mg$

$\frac{F_{sp} - \mu_k mg}{m} = \cancel{m}a$

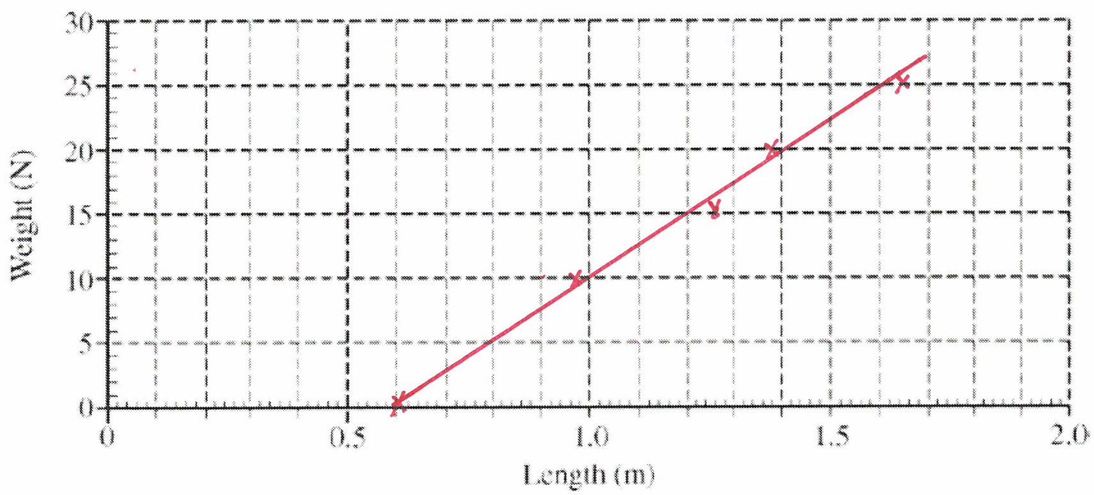
$\boxed{a = \frac{F_{sp}}{m} - \mu_k g}$

2) In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:



Weight (N)	0	10	15	20	25
Length (m)	0.60	0.97	1.24	1.37	1.64

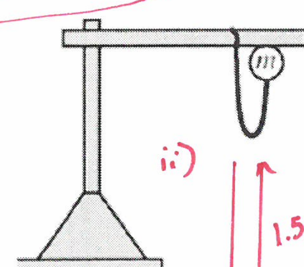
a) Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.



b) Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant k of the cord. $\text{Slope} = K = \frac{\Delta \text{Rise}}{\Delta \text{Run}} = \frac{25-10}{1.64-.97}$

$\boxed{K = 22.4\ \text{N/m}}$

if put in calculator graph) $\boxed{\text{Slope} = 24\ \text{N/m}}$



The student now attaches an object of unknown mass m to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as shown. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

c) Calculate the value of the unknown mass m of the object.

i) Determine the magnitude of the force in the cord when the when the mass reaches the equilibrium position.

ii) Determine the amount the cord has stretched when the mass reaches the equilibrium position.

iii) Calculate the speed of the object at the equilibrium position

iv) Is the speed in part iii above the maximum speed, explain your answer.

ii:) 1.5m, 0.6m unstretched, $\therefore \Delta x = 0.9\text{m}$

c) Ug KE Us — Ug KE Us

$U_g = U_{sp}$

$mgh = \frac{1}{2}kx^2$

$m = \frac{kx^2}{2gh} = \frac{(22.4\text{N})(0.9\text{m})^2}{2(9.8)(1.5\text{m})}$

$\boxed{m = 0.62\ \text{Kg}}$

i) Fsp, mg

$F_{net} = ma^0$

$F_{sp} - mg = 0$

$F_{sp} = mg = (0.62)(9.8)$

$\boxed{F_{sp} = 6.1\ \text{N}}$

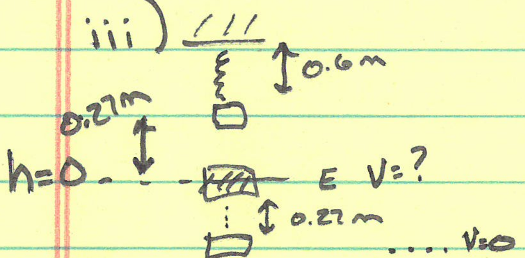
#2

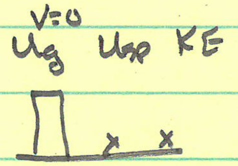
ⓒ ii) $F_{sp} = \Delta x K$

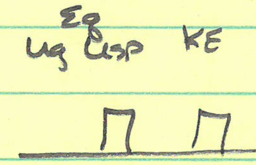
$\Delta x = \frac{F_{sp}}{K} = \frac{6.1N}{22.4 N/m}$

$\boxed{\Delta x = 0.27m}$

iii)







Equilibrium position

unstretched length + ΔX

= 0.60m + 0.27m

= 0.87m Eq X

$U_g = U_{sp} + KE$

$mgh = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$?

h = 0.87 m

K = 22.4 N/m

X = 0.27m

m = 0.62kg

$\frac{1}{2}mv^2 = mgh - \frac{1}{2}kx^2$

$= 2\left(\frac{\cancel{m}gh}{\cancel{m}} - \frac{\frac{1}{2}kx^2}{m}\right)$

$= 2\left(gh - \frac{kx^2}{2m}\right)$

$= 2\left[(9.8m/s^2)(0.87m) - \frac{(22.4N/m)(.27)^2}{0.62kg}\right]$

$v^2 = 2(8.53 - 2.63)$

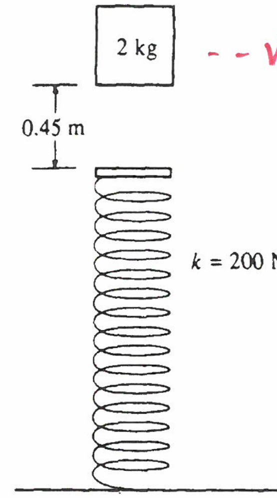
$\boxed{V = 3.4m/s}$

iv)

iv) This is the maximum speed because this is the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the mass down until it reaches its maximum compression and stops momentarily.

Pre-Exam - Unit 6 - FRQ - KEY

3) A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.



$- - V_0 = 0$

a) Determine the speed of the block at the instant it hits the end of the spring

2 ways

$V^2 = \cancel{V_0^2} + 2g(\Delta y)$

$V^2 = 2(9.8 m/s^2)(0.45 m)$ or

$V = 2.96 \approx 3 m/s$

$U_g \; KE \qquad U_g \; KE$

$U_g = KE$

$mgh = \frac{1}{2}\cancel{m}v^2$

$V^2 = 2gh$

$V = \sqrt{(2)(9.8 m/s)(0.45)}$

$V = 3 m/s$

b) Determine the force in the spring when the block reaches the equilibrium position

F_{sp}, mg

$F_{net} = m\cancel{a}^{0}$

$F_{sp} - mg = 0$

$F_{sp} = mg$

$= (2kg)(9.8 m/s^2)$

$F_s = 19.6 N$

c) Determine the distance that the spring is compressed at the equilibrium position

Δx?

$F_s = K\Delta x$

$\Delta x = \frac{F_s}{K} = \frac{19.6 N}{200 N/m}$

$\Delta x = 0.098 m$

d) Determine the speed of the block at the equilibrium position

0.45 m

$\Delta x = 0.098 m$

V_{max}? $h = 0$

Top: $U_g \; KE \; U_{sp}$

U_g

Equil: $U_g \; KE \; U_{sp}$

$KE + U_{sp}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$

$h = 0.45 + 0.098$

$\frac{1}{2}mv^2 = mgh - \frac{1}{2}kx^2$

$V^2 = 2\left[gh - \frac{Kx^2}{2m}\right]$

$= 2\left[(9.8)(0.548) - \frac{(200)(0.098)^2}{2(2kg)}\right]$

$V^2 = 2(5.37 - 0.48)$

$\boxed{V = 3.1 m/s}$

e) Determine the resulting amplitude of the oscillation that ensues

A?

$V = 0$, U_{sp} max

Equil: V_{max}, $U_{sp} = 0$

$\frac{1}{\cancel{2}}Kx^2 = \frac{1}{\cancel{2}}mv^2$

$A^2 = \frac{mv^2}{K}$

$A^2 = \frac{(3.1 m/s)^2(2kg)}{200 N/m}$

$\boxed{A = 0.31 m}$

f) Is the speed of the block a maximum at the equilibrium position, explain.

f) This is the maximum speed because this was the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the box down until it reaches its maximum compression and stops momentarily.

g) Determine the period of the simple harmonic motion that ensues

$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{2kg}{200 N/m}}$

$\boxed{T = 0.63 s}$