## Pendulum Periods

A swinging pendulum keeps a very regular beat. It is so regular, in fact, that for many years the pendulum was the heart of clocks used in astronomical measurements at the Greenwich Observatory.

There are at least three things you could change about a pendulum that might affect the period (the time for one complete cycle):

- the amplitude of the pendulum swing
- the length of the pendulum, measured from the center of the pendulum bob to the point of support
- the mass of the pendulum bob

To investigate the pendulum, you need to do a controlled experiment; that is, you need to make measurements, changing only one variable at a time. Conducting controlled experiments is a basic principle of scientific investigation.

In this experiment, you will use a Photogate capable of about 4 ms precision to measure the period of one complete swing of a pendulum. By conducting a series of controlled experiments with the pendulum, you can determine how each of these quantities affects the period.


Figure 1

## OBJECTIVES

- Measure the period of a pendulum as a function of amplitude.
- Measure the period of a pendulum as a function of length.
- Measure the period of a pendulum as a function of bob mass.


## MATERIALS

LabQuest
LabQuest App
Vernier Photogate
protractor
string

2 ring stands and pendulum clamp
masses of $100 \mathrm{~g}, 200 \mathrm{~g}, 300 \mathrm{~g}$
meter stick
Logger Pro software (optional)

## PRELIMINARY QUESTIONS

1. Make a pendulum by tying a 100 cm string to a mass. Hold the string in your hand and let the mass swing. Observing only with your eyes, does the period depend on the length of the string? Does the period depend on the amplitude of the swing?
2. Try a different mass on your string. Does the period seem to depend on the mass?

## PROCEDURE

1. Use the ring stand to hang the 200 g mass from two strings. Attach the strings to a horizontal rod about 10 cm apart, as shown in Figure 1. This arrangement will let the mass swing only along a line, and will prevent the mass from striking the Photogate. The length of the pendulum is the distance from the point on the rod halfway between the strings to the center of the mass. The pendulum length should be at least 100 cm .
2. Attach the Photogate to the second ring stand. Position it so that the mass blocks the Photogate while hanging straight down.
3. Connect the Photogate to DIG 1 of LabQuest and choose New from the File menu. If you have an older sensor that does not auto-ID, manually set up the sensor.
4. Set up LabQuest for data-collection with a pendulum.
a. On the Meter screen, tap Mode.
b. Change Photogate Mode to Pendulum Timing and select OK.
5. Temporarily hold the mass out of the center of the Photogate. Observe the live readings on the screen. Block the Photogate 1 with your hand. Note that the Photogate is shown as Blocked on the screen. Remove your hand and the display should change to Unblocked. Repeat for Photogate 2.
6. Temporarily hold the mass out of the center of the Photogate. Start data collection to prepare the Photogate.
7. Now you can perform a trial measurement of the period of your pendulum. Hold the mass from about $10^{\circ}$ from vertical and release. (For a pendulum that is 100 cm long, that corresponds to pulling the bob only about 15 cm to the side.) After five trials have been recorded, stop data collection.
8. Choose Statistics from the Analyze menu. Record the Average Period in your data table.
9. When you are ready measure another period, simply start data collection again. You will use this method for each period measurement below.

## Part I Amplitude

10. Determine how the period depends on amplitude. Measure the period for five different amplitudes. Use a range of amplitudes, from just barely enough to unblock the Photogate, to about $30^{\circ}$. Each time, measure the amplitude using the protractor so that the mass with the string is released at a known angle. Repeat this step for each different amplitude. Record the data in your data table.

## Part II Length

11. Use the method you learned above to investigate the effect of changing pendulum length on the period. Use the 200 g mass and a consistent amplitude of $10^{\circ}$ for each trial. Vary the pendulum length in steps of 10 cm , from 100 cm to 50 cm . If you have room, continue to a longer length (up to 200 cm ). Repeat Steps 7-9 for each length. Record the data in the second data table below. Measure the pendulum length from the rod to the middle of the mass.

## Part III Mass

12. Use the three masses to determine if the period is affected by changing the mass. Measure the period of the pendulum constructed with each mass, taking care to keep the distance from the ring stand rod to the center of the mass the same each time, as well as keeping the amplitude the same. Repeat Steps 7-9 for each mass, using an amplitude of about $10^{\circ}$. Record the data in your data table.

## DATA TABLE

## Part I Amplitude

| Amplitude <br> $\left({ }^{\circ}\right)$ | Average period <br> $(\mathrm{s})$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Part II Length

| Length <br> $(\mathrm{cm})$ | Average period <br> $(\mathrm{s})$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Part III Mass

| Mass <br> $(\mathrm{g})$ | Average period <br> (s) |
| :---: | :---: |
|  |  |
|  |  |
|  |  |

## ANALYSIS

1. In measuring the pendulum period, should the interface measure the time between two adjacent blocks of the photogate? Or is some other measurement logic used? Why?
2. Using LabQuest, Logger Pro software, or graph paper, plot a graph of pendulum period $T v s$. amplitude in degrees. Scale each axis from the origin ( 0,0 ). According to your data, does the period depend on amplitude? Explain.
3. Using your LabQuest, Logger Pro, or graph paper, plot a graph of pendulum period $T v s$. length $\ell$. Scale each axis from the origin $(0,0)$. Does the period appear to depend on length?
4. Using your LabQuest, Logger Pro, or graph paper, plot the pendulum period vs. mass. Scale each axis from the origin ( 0,0 ). Does the period appear to depend on mass? Do you have enough data to answer this conclusively?
5. To examine more carefully how the period $T$ depends on the pendulum length $\ell$, create the following two additional graphs of the same data: $T^{2} v s$. $\ell$ and $T v s$. $\ell^{2}$. Of the three periodlength graphs, which is closest to a direct proportion; that is, which plot is most nearly a straight line that goes through the origin?
6. Using Newton's laws, we could show that for a simple pendulum the period $T$ is related to the length $\ell$ and free-fall acceleration $g$ by

$$
T=2 \pi \sqrt{\frac{\ell}{g}}, \text { or } T^{2}=\left(\frac{4 \pi^{2}}{g}\right) \times \ell
$$

Does one of your graphs support this relationship? Explain. (Hint: Can the term in parentheses be treated as a constant of proportionality?)

## EXTENSIONS

1. From your graph of $T^{2} v s$. $\ell$, determine a value for $g$.
2. Given what you observed in this experiment, write a set of rules for constructing a pendulum clock that is reliable under a variety of temperatures.
3. Try a larger range of amplitudes than you used in Part I. If you did not see a change in period with amplitude before, you should now. Check a college physics textbook for an expression for the period of a pendulum at large amplitudes and compare to your own data.
4. Use an air table and air table puck as your pendulum. Tip the air table to a variety of angles, $\theta$, and determine the relationship between the period and the angle.
