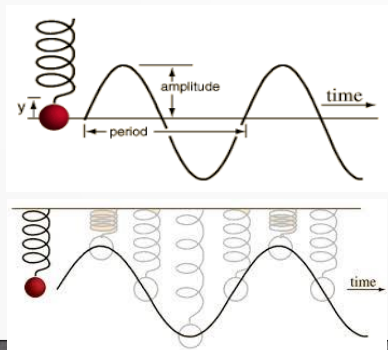
SHM: Mass-Spring System

Period of a Mass-Spring System depends on the **mass** and on the **spring constant**. (amplitude does not affect)

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$



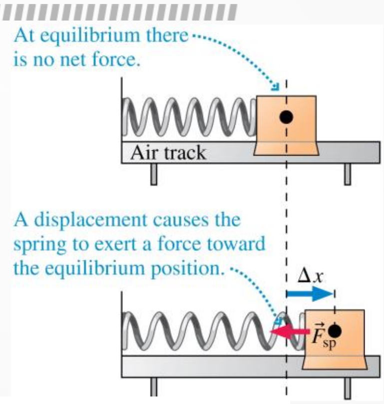
- Each spring has an associated **spring constant K**, which measures how "tight" the spring is

Hooke's Law – relationship between force and displacement

$$F_{elastic} = -kx$$

Linear Restoring Forces and SHM

- If we displace a glider attached to a spring from its equilibrium position, the spring exerts a restoring force back toward equilibrium.
- This is a **linear restoring force**; **the net force is toward the equilibrium position and is proportional to the distance from equilibrium.**

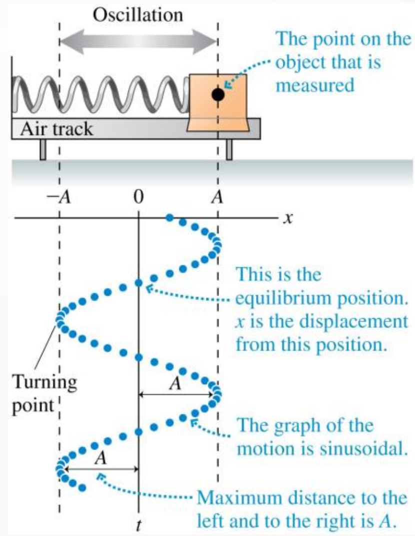


$$(F_{net})_x = -kx$$

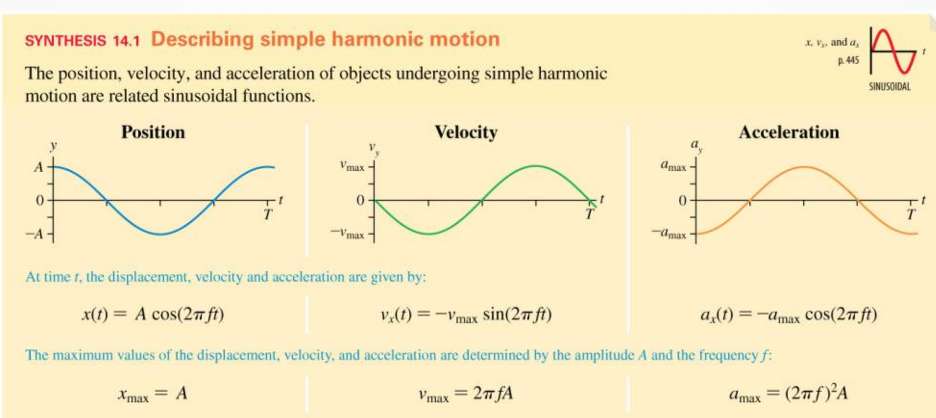
The negative sign tells us that this is a restoring force because the force is in the direction opposite the displacement. If we pull the glider to the right (x is positive), the force is to the left (negative)—back toward equilibrium.

Motion of a Mass on a Spring

- The **amplitude *A*** is the object's maximum displacement from equilibrium.
- **Oscillation about an equilibrium position with a linear restoring force is always simple harmonic motion.**



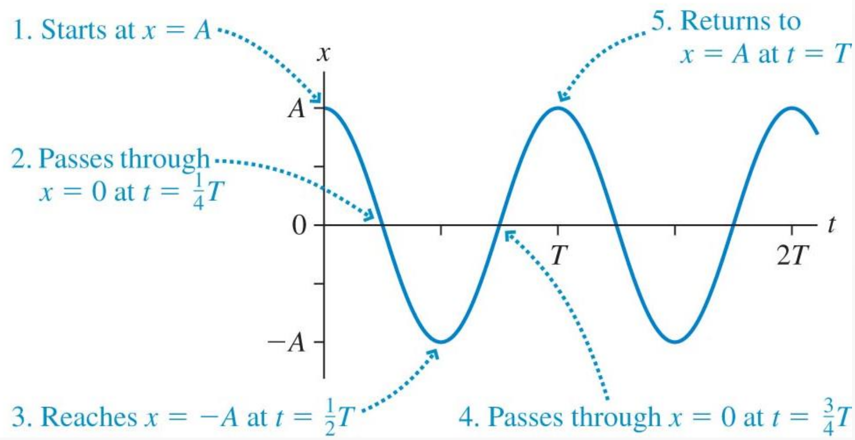
SHM Motion Graphs



Can you draw where the SHM is in relationship to the graphs?

Simple Harmonic Motion

The position-versus-time graph for simple harmonic motion.



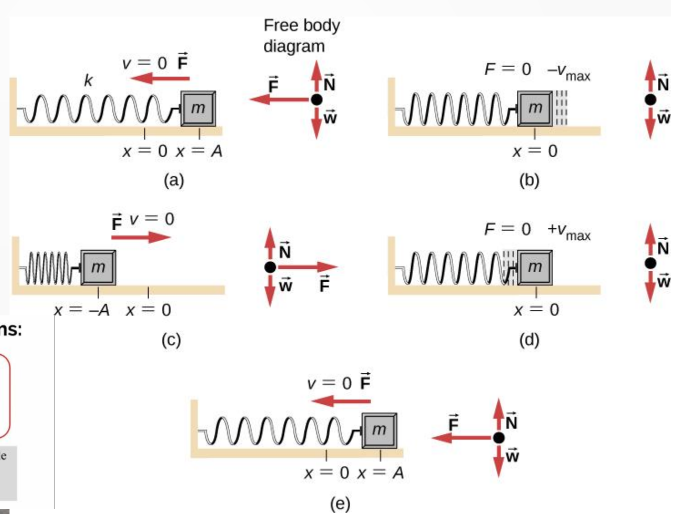
SHM: Finding the Vmax, Spring system

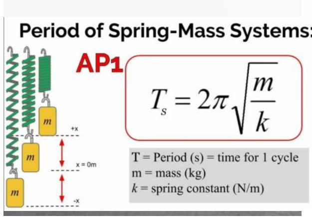
- Because of conservation of energy, the maximum potential energy must be equal to the maximum kinetic energy:

$$\frac{1}{2}m(v_{\max})^2 = \frac{1}{2}kA^2$$

Solving for the maximum velocity

$$v_{\max} = \sqrt{\frac{k}{m}}A$$





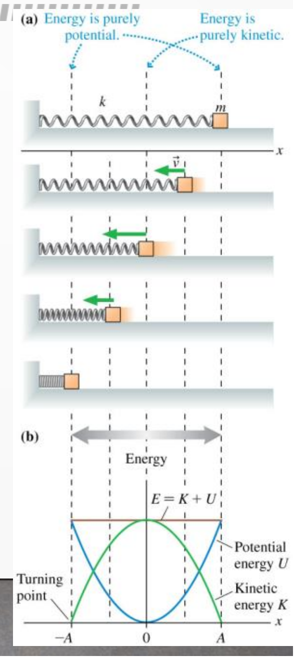
Energy in SHM: Linear Spring

- At maximum displacement, the energy is purely potential:

$$E(\text{at } x = \pm A) = U_{\max} = \frac{1}{2}kA^2$$

- At x = 0, the equilibrium position, the energy is purely kinetic:

$$E(\text{at } x = 0) = K_{\max} = \frac{1}{2}m(v_{\max})^2$$

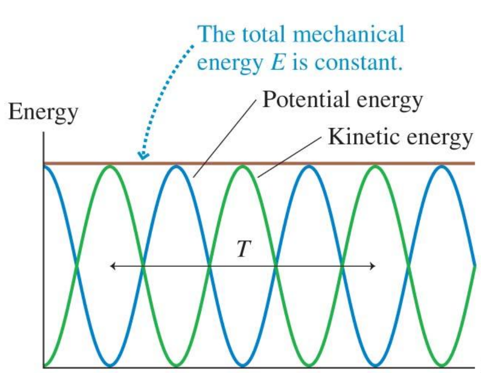


Energy in Simple Harmonic Motion

Energy is conserved in SHM.

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(v_{\max})^2 \quad \text{(conservation of energy)}$$

FIGURE 14.11 Kinetic energy, potential energy, and the total mechanical energy for simple harmonic motion.

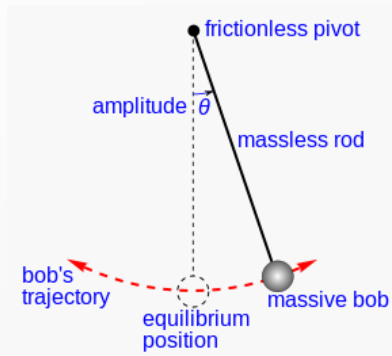


SHM: Simple Pendulum

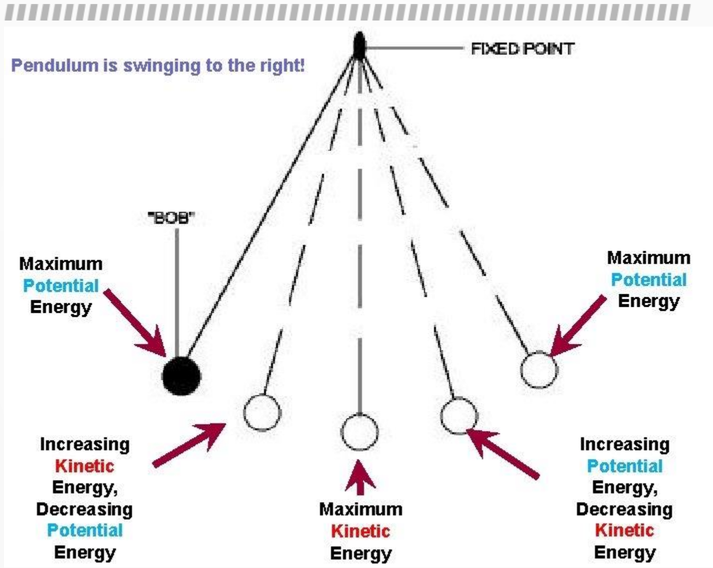
- **Period of simple Pendulum** depends on the **length** and the **free-fall acceleration**

$$T_p = 2\pi\sqrt{\frac{L}{g}}$$

- The period **does not** depend on the mass of the bob or on the amplitude (for small angles)



Energy in SHM: Pendulum



Finding the Frequency for SHM

- Because of conservation of energy, the maximum potential energy must be equal to the maximum kinetic energy:

$$\frac{1}{2}m(v_{\max})^2 = \frac{1}{2}kA^2$$

- Solving for the maximum velocity we find

- Earlier we found that

$$v_{\max} = 2\pi f A$$