

## SHM: Mass-Spring System

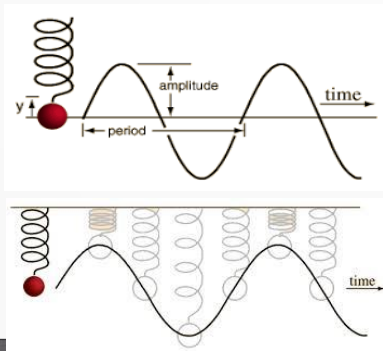
**Period** of a Mass-Spring System depends on the **mass** and on the **spring constant**. (amplitude does not affect)

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

- Each spring has an associated **spring constant** **K**, which measures how "tight" the spring is

**Hooke's Law** – relationship between force and displacement

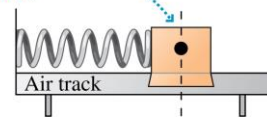
$$F_{\text{elastic}} = -kx$$



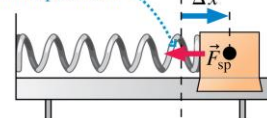
## Linear Restoring Forces and SHM

- If we displace a glider attached to a spring from its equilibrium position, the spring exerts a restoring force back toward equilibrium.
- This is a **linear restoring force**; the **net force is toward the equilibrium position and is proportional to the distance from equilibrium**.

At equilibrium there is no net force.



A displacement causes the spring to exert a force toward the equilibrium position.

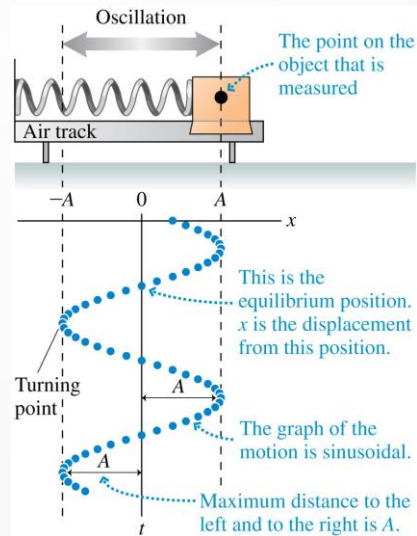


$$(F_{\text{net}})_x = -kx$$

The negative sign tells us that this is a restoring force because the force is in the direction opposite the displacement. If we pull the glider to the right ( $x$  is positive), the force is to the left (negative)—back toward equilibrium.

## Motion of a Mass on a Spring

- The **amplitude  $A$**  is the object's maximum displacement from equilibrium.
- Oscillation about an equilibrium position with a linear restoring force is always simple harmonic motion.**



## SHM Motion Graphs

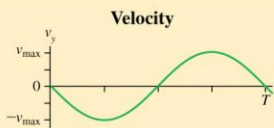
### SYNTHESIS 14.1 Describing simple harmonic motion

The position, velocity, and acceleration of objects undergoing simple harmonic motion are related sinusoidal functions.

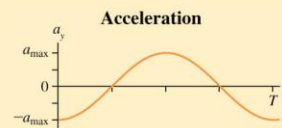
$x$ ,  $v_x$ , and  $a_x$   
p. 445  
SINUSOIDAL



$$x(t) = A \cos(2\pi ft)$$



$$v_x(t) = -v_{\max} \sin(2\pi ft)$$



$$a_x(t) = -a_{\max} \cos(2\pi ft)$$

The maximum values of the displacement, velocity, and acceleration are determined by the amplitude  $A$  and the frequency  $f$ :

$$x_{\max} = A$$

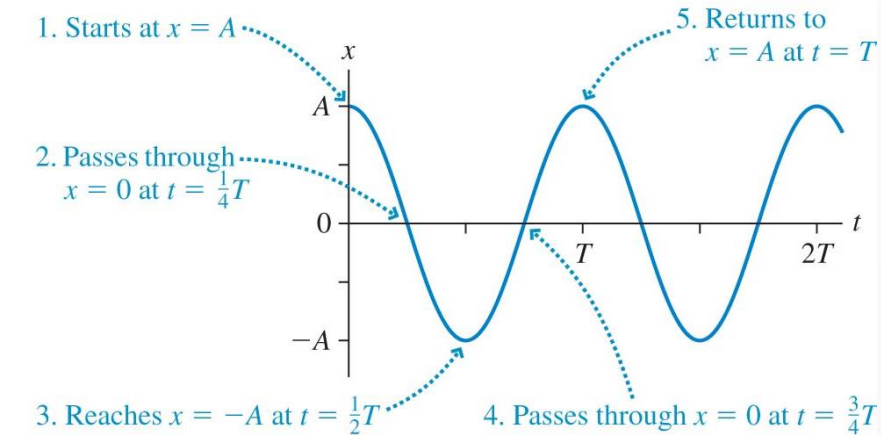
$$v_{\max} = 2\pi fA$$

$$a_{\max} = (2\pi f)^2 A$$

Can you draw where the SHM is in relationship to the graphs?

# Simple Harmonic Motion

The position-versus-time graph for simple harmonic motion.



## SHM: Finding the Vmax, Spring system

- Because of conservation of energy, the maximum potential energy must be equal to the maximum kinetic energy:

$$\frac{1}{2}m(v_{\max})^2 = \frac{1}{2}kA^2$$

Solving for the maximum velocity

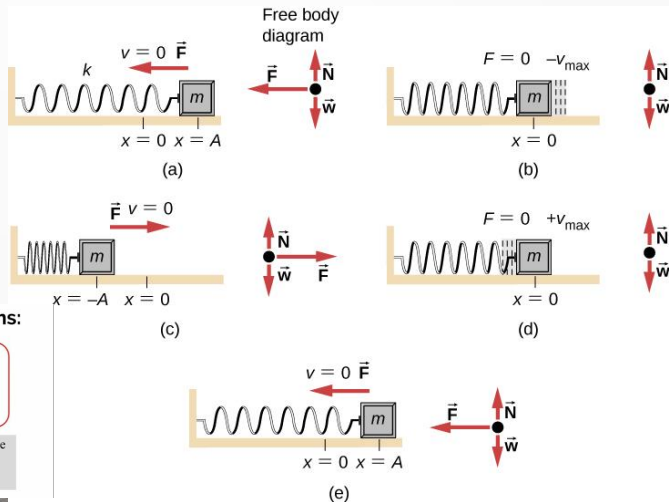
$$v_{\max} = \sqrt{\frac{k}{m}}A$$

Period of Spring-Mass Systems:

**AP1**

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

T = Period (s) = time for 1 cycle  
 m = mass (kg)  
 k = spring constant (N/m)



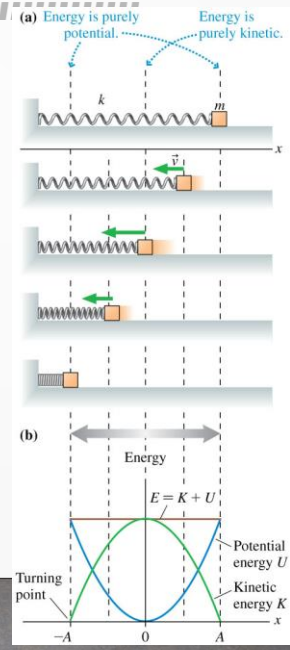
## Energy in SHM: Linear Spring

- At maximum displacement, the energy is purely potential:

$$E(\text{at } x = \pm A) = U_{\text{max}} = \frac{1}{2}kA^2$$

- At  $x = 0$ , the equilibrium position, the energy is purely kinetic:

$$E(\text{at } x = 0) = K_{\text{max}} = \frac{1}{2}m(v_{\text{max}})^2$$

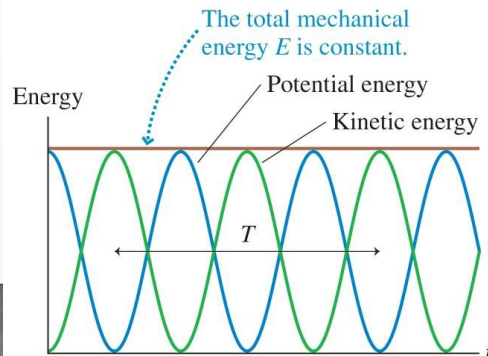


## Energy in Simple Harmonic Motion

Energy is conserved in SHM.

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(v_{\text{max}})^2 \quad (\text{conservation of energy})$$

**FIGURE 14.11** Kinetic energy, potential energy, and the total mechanical energy for simple harmonic motion.

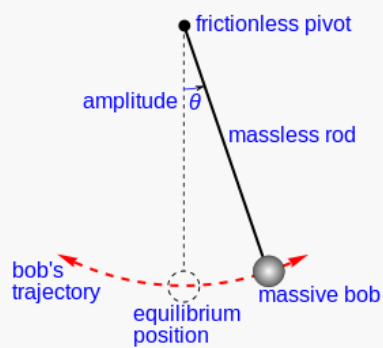


# SHM: Simple Pendulum

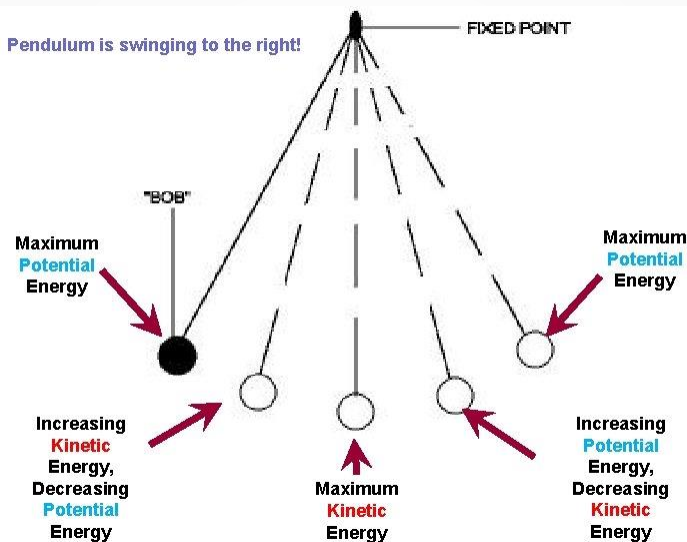
- **Period of simple Pendulum** depends on the **length** and the **free-fall acceleration**

$$T_p = 2\pi \sqrt{\frac{L}{g}}$$

- The period **does not** depend on the mass of the bob or on the amplitude (for small angles)



# Energy in SHM: Pendulum



## Finding the Frequency for SHM

- Because of conservation of energy, the maximum potential energy must be equal to the maximum kinetic energy:

$$\frac{1}{2}m(v_{\max})^2 = \frac{1}{2}kA^2$$

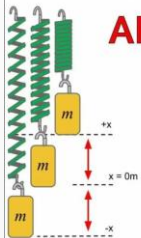
- Solving for the maximum velocity we find

- Earlier we found that

$$v_{\max} = 2\pi fA$$

**Period of Spring-Mass Systems:**

**AP1**


$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

T = Period (s) = time for 1 cycle  
m = mass (kg)  
k = spring constant (N/m)