Momentum and Collisions

Physics Chapter 6

Objectives

- **Compare** the momentum of different moving objects.
- **Compare** the momentum of the same object moving with different velocities.
- **Identify** examples of change in the momentum of an object.
- **Describe** changes in momentum in terms of force and time

Momentum

- **Momentum (p)** can be defined as "mass in motion." All objects have mass; so if an object is moving, then it has momentum
  - Momentum depends: mass and velocity
  - Vector Quantity – it has magnitude and direction
  - **p = mass x velocity**
    - p=mv
    - Units – Kg m/s

- Can **smaller** objects ever have as much momentum as a **large** object?
  - Yes, but smaller mass object has to move with a higher velocity

Impulse

Given that … **Momentum = *mv***

If velocity changes, momentum changes, and acceleration (either + or –) occurs

But we know:

1. For acceleration to occur, a force has to be applied.
2. If a given force is applied over a longer time, more acceleration occurs.

IMPULSE is a measure of how much force is applied for how much time, and it's equal to the change in momentum.
or

- Is the change in momentum, and results from force acting over a period of time.

A force applied over time will change the momentum of an object:

Impulse

- **Impulse (J)** = Change in momentum = Force x time
  - **J** = F Δt
  - Units: N·s or Kg·m/s

- So to achieve any particular change in momentum, you can either have a

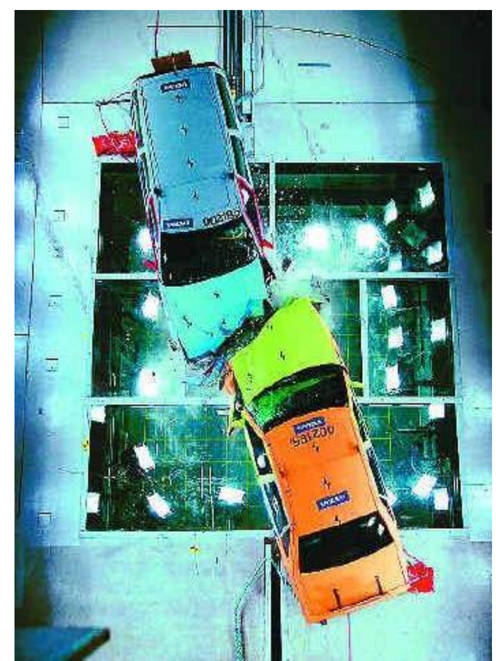
  large force x **small time**

  or

  small force x **large time**

This explains how crumple zones work!

- Volvo cars love collisions so much that they have crumple zones which make the collisions last longer. A longer time means that you can achieve the same change in momentum during a collision and disperse a smaller force. And it is the force that hurts the passengers.



Impulse changes Momentum

A greater impulse exerted on an object $\Rightarrow$ A greater change in momentum

OR

Impulse = Change in momentum

OR

Impulse = $\Delta(mv)$

Impulse can be exerted on an object to either **INCREASE** or **DECREASE** its momentum.

Decreasing Momentum

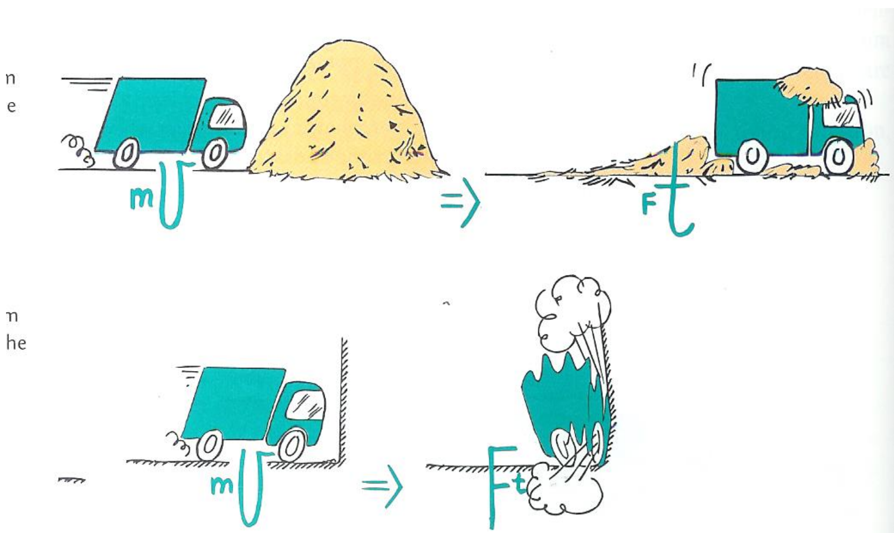
It takes an impulse to change momentum, and

Remember … Impulse = F x t

If you want to stop something's motion, you can apply a LOT of force over a short time,

Or, you can apply a little force over a longer time.

Remember, things BREAK if you apply a lot of force to them.



Impulse – Momentum Theorem

- The impulse due to all forces acting on an object (the net force) is equal to the change in momentum of the object:

$$\mathbf{F}\Delta t = \Delta\mathbf{p} = m\mathbf{v}_\mathbf{f} - m\mathbf{v}_\mathbf{i}$$

- Explains follow through in sports:
  - longer contact $\Rightarrow$ greater change in momentum
- Force is reduced when the time interval of an impact is increased

Impulse - Momentum Example

A 1.3 kg ball is coming straight at a 75 kg soccer player at 13 m/s who kicks it in the exact opposite direction at 22 m/s with an average force of 1200 N. How long are his foot and the ball in contact?

answer:

$F_{net}\, t = \Delta p.$

$\Delta p = m \Delta v = m (v_f - v_0)$ Since the ball changes direction

$\Delta p = 1.3\text{kg}\,[22\text{m/s} - (-13\text{m/s})]$

$\Delta p = 46\ \text{kg}\cdot\text{m /s}$

$(1200\text{N})\ t = 46\ \text{kg}\cdot\text{m /s}.$

$t = 0.038\ \text{s}$

During this contact time the ball compresses substantially and then decompresses. This happens too quickly for us to see, though. This compression occurs in many cases, such as hitting a baseball or golf ball.

Conservation of Momentum

The Law of Conservation of Momentum:

The total momentum of all objects interacting with one another remains constant regardless of the nature of the forces between the objects.

$$m_1\mathbf{v}_{\mathbf{1,i}} + m_2\mathbf{v}_{\mathbf{2,i}} = m_1\mathbf{v}_{\mathbf{1,f}} + m_2\mathbf{v}_{\mathbf{2,f}}$$

total initial momentum = total final momentum

- In all interactions between isolated objects, momentum is conserved

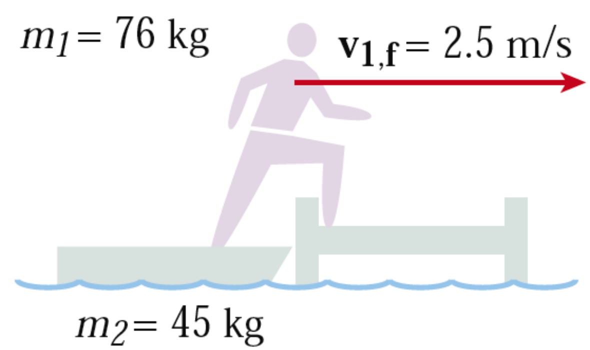
  Or

- In the absences of an external force, the momentum of a system remains unchanged

Conservation of Momentum Problem

Example: A 76 kg boater, initially at rest in a stationary 45 kg boat, steps out of the boat and onto the dock. If the boater moves out of the boat with a velocity of 2.5 m/s to the right, what is the final velocity of the boat?

Diagram:



Given:

<u>Man</u>

$m_{man} = 76$ kg

$v_{man,i} = 0$

$v_{man,f} = 2.5$ m/s (right)

<u>Boat</u>

$m_{Boat} = 45$ kg

$v_{Boat,i} = 0$

$v_{Boat,f} = ?$

Conservation of Momentum Problem

Both man and Boat start from rest

Soln:

$$m_{man}\cancelto{0}{V_{man,i}} + m_{Boat}\cancelto{0}{V_{Boat,i}} = m_{man}V_{man,f} + m_{Boat}\mathbf{V}_{\mathbf{Boat,f}}$$

$$m_{Boat}\mathbf{V}_{\mathbf{Boat,f}} = - m_{man}V_{man,f}$$

$$\mathbf{V}_{\mathbf{Boat,f}} = - (m_{man} / m_{Boat})\, V_{man,f}$$

$$\mathbf{V}_{\mathbf{Boat,f}} = - (76kg / 45kg)\,(2.5\ m/s)$$

$$\boxed{\begin{array}{l}\mathbf{V}_{\mathbf{Boat,f}} = - 4.2\ m/s \\ \quad\quad \text{or } 4.2\ m/s \text{ to the left}\end{array}}$$

Types of Collisions

Objectives

- **Identify** different types of collisions.
- **Determine** the changes in kinetic energy during perfectly inelastic collisions.
- **Compare** conservation of momentum and conservation of kinetic energy in perfectly inelastic and elastic collisions.
- **Find** the final velocity of an object in perfectly inelastic and elastic collisions

Types of Collisions

The law of conservation of momentum is neatly seen in collisions.
Net momentum (before collisions) = **Net momentum** (after collisions)

1. **Elastic Collisions**
   - **KE is conserved**
   - **All momentum is conserved**
   - When a Ball hits the ground and bounces to the same height, the collision is elastic

2. **Perfectly Inelastic Collisions**
   - **KE is converted into other forms of energy**
   - **All momentum is conserved**
   - When a Ball hits the ground and sticks, the collision would be totally inelastic

3. **Inelastic Collisions**
   - you can't say anything about the speed at which they leave each other without doing a calculation

Elastic Collisions

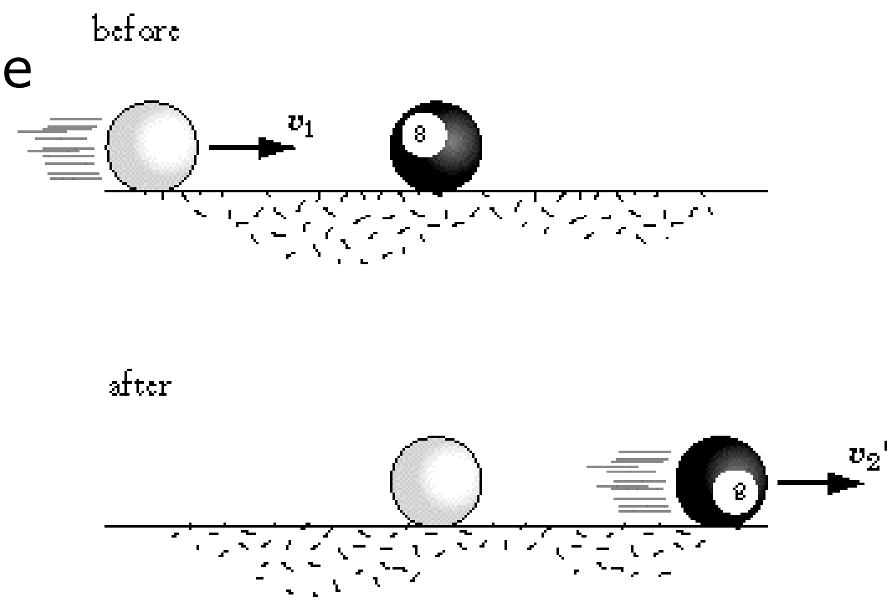
<u>Elastic</u> – 2 objects collide – move separately afterward

- Total momentum and **KE remain constant**
  - No heat generation
  - Objects maintain original shape

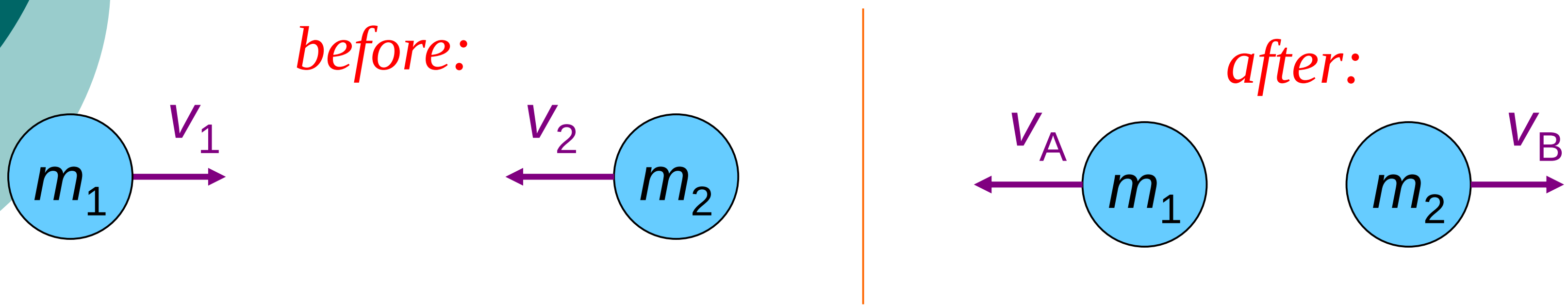
- $M_1V_{1i} + M_2V_{2i} = M_1V_{1f} + M_2V_{2f}$

- **Example:** Two billiard balls collide

- Moving billiard ball hits another billiard ball at rest, head on. First ball comes to rest and 2nd ball moves with the initial velocity of second ball.



Elastic Collisions



conservation of momentum:

$$m_1 v_1 - m_2 v_2 = -m_1 v_A + m_2 v_B$$

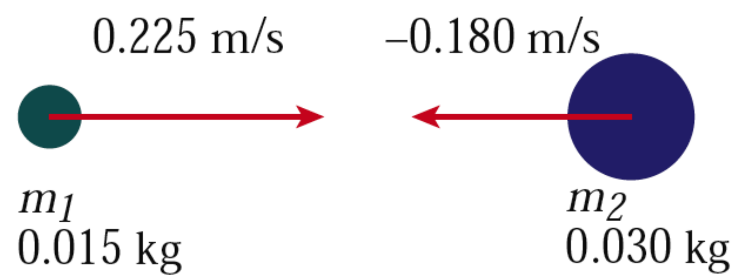
conservation of energy:

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 v_A^2 + \tfrac{1}{2} m_2 v_B^2$$

Elastic Collision - Example

A 0.015 kg marble moving to the right at 0.225 m/s makes an elastic head-on collision with a 0.030 kg shooter marble moving to the left at 0.180 m/s. After the collision, the smaller marble moves to the left at 0.315 m/s. Assume that neither marble rotates before or after the collision and that both marbles are moving on a frictionless surface. What is the velocity of the 0.030 kg marble after the collision?

Diagram:



Given:

m_1 = 0.015 kg

$\mathbf{v_{1i}}$ = 0.225 m/s to the right

$\mathbf{v_{1f}}$ = 0.315 m/s to the left

m_2 = 0.030 kg

$\mathbf{v_{2i}}$ = 0.180 m/s to the left

$\mathbf{v_{2f}}$ = ?

Elastic Collision - Example

Given:

$$m_1\mathbf{v_{1i}} + m_2\mathbf{v_{2i}} = m_1\mathbf{v_{1f}} + m_2\overset{\mathbf{?}}{\mathbf{v_{2f}}}$$

$$\mathbf{v_{2f}} = \left(m_1\mathbf{v_{1i}} + m_2\mathbf{v_{2i}} - m_1\mathbf{v_{1f}}\right)/m_2$$

$$v_{2,f} = \frac{(0.015\text{ kg})(0.225\text{ m/s}) + (0.030\text{ kg})(-0.180\text{ m/s}) - (0.015\text{ kg})(-0.315\text{ m/s})}{0.030\text{ kg}}$$

$$v_{2,f} = \frac{\left(3.4\times10^{-3}\text{ kg}\cdot\text{m/s}\right) + \left(-5.4\times10^{-3}\text{ kg}\cdot\text{m/s}\right) - \left(-4.7\times10^{-3}\text{ kg}\bullet\text{m/s}\right)}{0.030\text{ kg}}$$

$$v_{2,f} = \frac{2.7\times10^{-3}\text{ kg}\cdot\text{m/s}}{3.0\times10^{-2}\text{ kg}}$$

$$\boxed{\mathbf{v_{2,f}} = 9.0\times10^{-2}\text{ m/s to the right}}$$

Perfectly Inelastic Collisions

- Two objects collide sticking together
- Become stuck together and travel as a single unit after collision
- Momentum is conserved and **KE is lost**
  - Ignore external forces (heat gain or lost, sound generation)
  - Results the velocity of the 2 colliding objects is the same after they collide

- $M_1V_{1i} + M_2V_{2i} = V_f\,(M_1 + M_2)$
- $\Delta KE = KE_f - KE_i$

- **Other Examples:**
- Freight train cars collide (neglect noise)
- Two snowballs collide and stick together

Example Problem: Perfectly Inelastic Collisions

Two clay balls collide head-on in a perfectly inelastic collision. The first ball has a mass of 0.500 kg and an initial velocity of 4.00 m/s to the right. The second ball has a mass of 0.250 kg and an initial velocity of 3.00 m/s to the left. What is the decrease in kinetic energy during the collision?

Given:

ΔKE = ? ($\Delta KE = KE_f - KE_i$)

$m_1 = 0.500$ kg

$\mathbf{v_{1i}}$ = 4.00 m/s to the right

$V_{1f} = V_{2f}$

$KE_{1i} = 1/2 m_1 V_{1i}^2$

m2 = 0.250 kg

$\mathbf{v_{2i}}$ = 3.00 m/s to the left

$KE_{2i} = 1/2 m_2 V_{2i}^2$

$KE_f = 1/2(m_1 + m_2)\mathbf{V_f}^2$

Need V_f before you can solve for ΔKE

Example Problem: Perfectly Inelastic Collisions

Soln: Find V_f 1st

$M_1V_{1i} + M_2V_{2i} = V_f (M_1 + M_2)$

$\mathbf{V_f =} M_1V_{1i} + M_2V_{2i}/(M_1+M_2)$

V_f =(.500kg)(4.00 m/s)+(.250kg)(-3.00m/s) / (.500kg+.250kg)

V_f = 1.67 m/s to the right

$KE_i = KE_{1i} + KE_{2i} = ½\, m_1V_{1i}^2 + ½m_2V_{2i}^2$

KE_i = ½ (.500kg)(4.00m/s)^2 + ½ (.250kg)(-3.00 m/s)^2

KEi= 5.12 J

$KE_f = ½\,(m_1+m_2)V_f^2$

KE_f = ½ (.500 kg +4.00 m/s)(1.67 m/s)^2

KE_f = 1.05 J

$\boxed{\begin{aligned} \Delta KE &= KE_f - KE_i \\ \Delta KE &= 1.05\text{ J} - 5.12\text{ J} \\ \Delta KE &= -4.07\text{ J} \end{aligned}}$

Extra Information: Equivalent Momentum

Car: $m = 1800$ kg; $v = 80$ m/s

$p = 1.44 \cdot 10^5$ kg$\cdot$m/s

Bus: $m = 9000$ kg; $v = 16$ m/s

$p = 1.44 \text{x} 10^5$ kg$\cdot$m/s

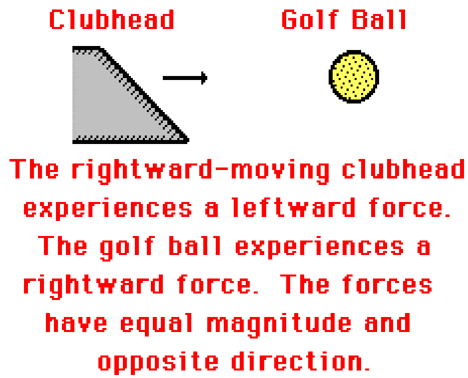
Train: $m = 3.6 \text{x} 10^4$ kg; $v = 4$ m/s

$p = 1.44 \cdot 10^5$ kg$\cdot$m/s

Case 1: Increasing Momentum

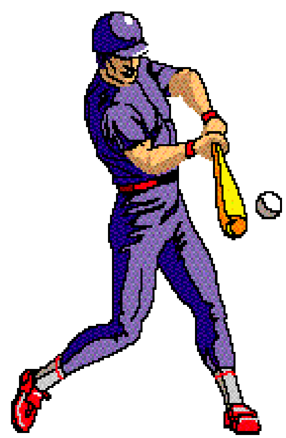
Examples:

Hitting a golf ball:



Apply the greatest force possible for the longest time possible.
Accelerates the ball from 0 to high speed in a very short time.

Baseball and bat:

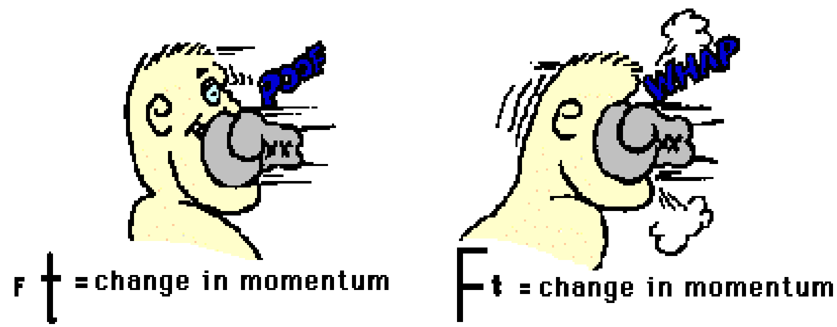


The impulse of the bat decelerates the ball and accelerates it in the opposite direction very quickly.

Case 2: Decreasing Momentum over a Short Time

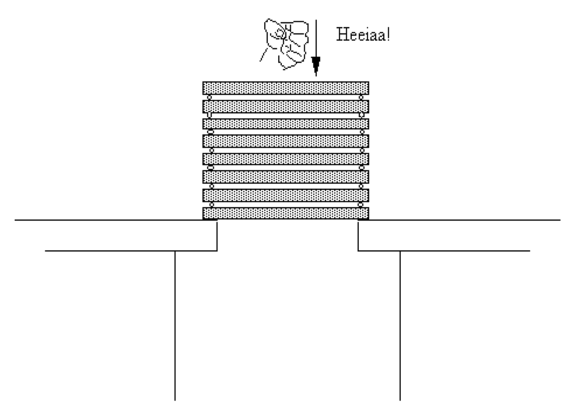
If the boxer moves away from the punch, he extends the time and decreases the force while stopping the punch.

If he moves toward the punch, he decreases the time and increases the force



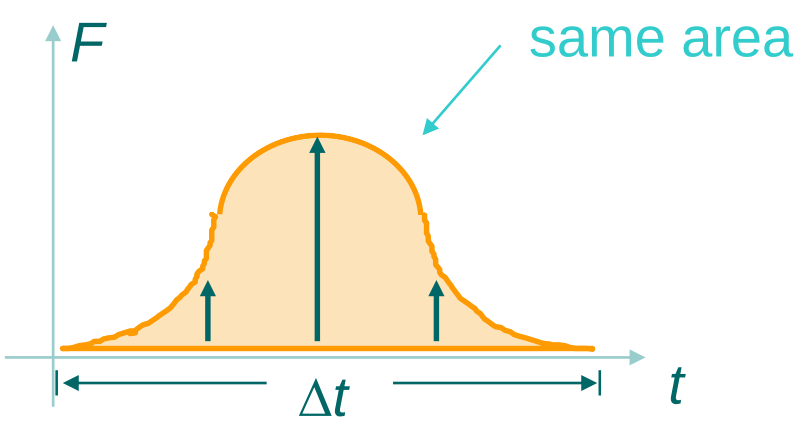
The airbag extends the time over which the impulse is exerted and decreases the force.

Hitting the bricks with a sharp karate blow very quickly maximizes the force exerted on the bricks and helps to break them.

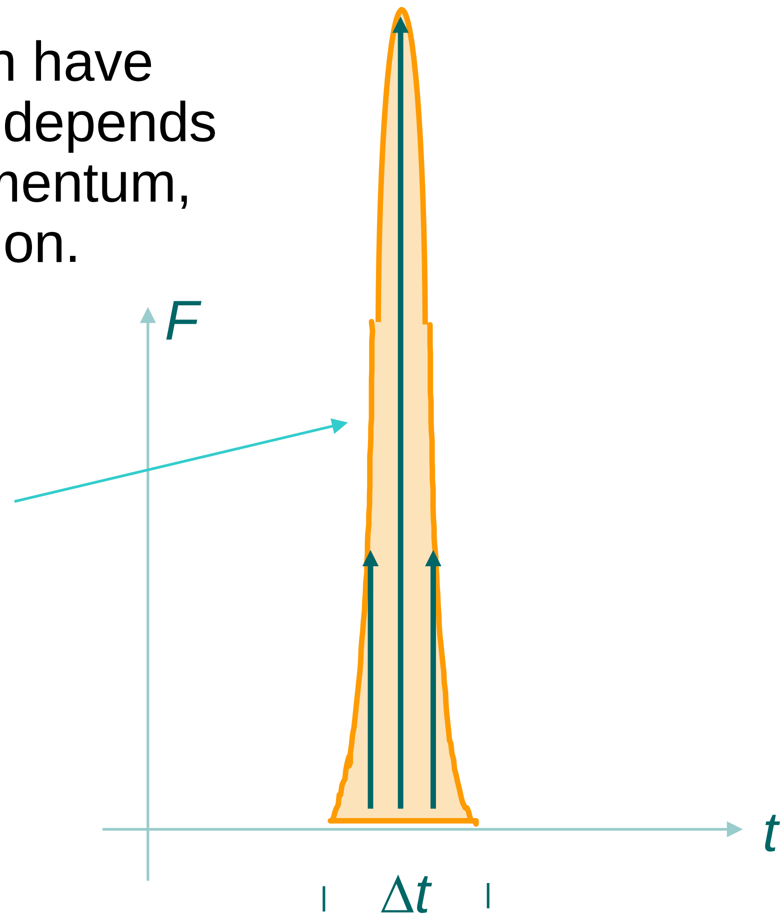


Extra: Force and Impulse

- Two different collisions can have the same impulse since ***P*** depends only on the change in momentum, not the nature of the collision.



Δt big, ***F*** small



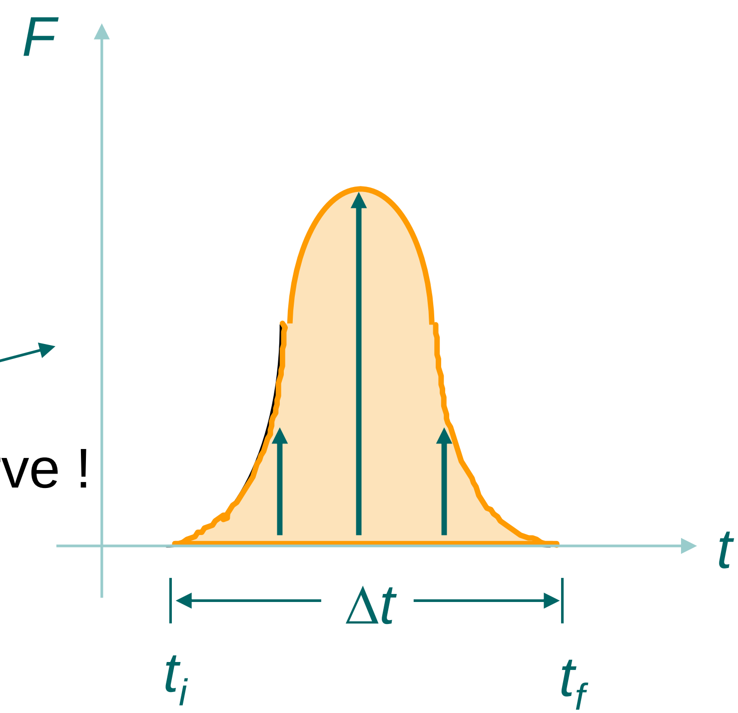
Δt small, ***F*** big

Extra: Force and Impulse

- The diagram shows the force vs time for a typical collision. The impulse, ***P***, of the force is a vector defined as the integral of the force during the collision.

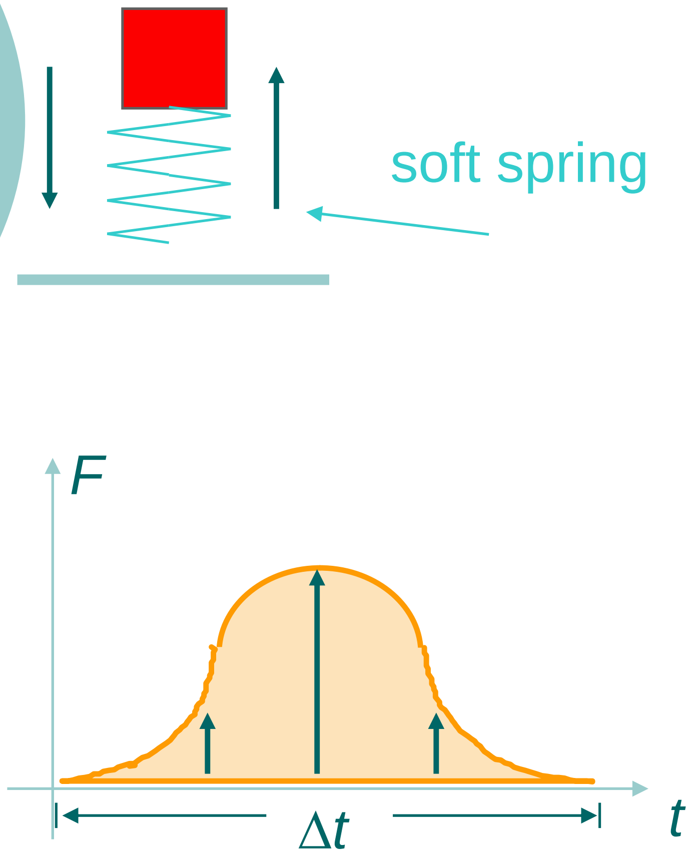
$$\boldsymbol{P} = \int^{t} \boldsymbol{F}\, dt$$

Impulse ***P*** = area under this curve !

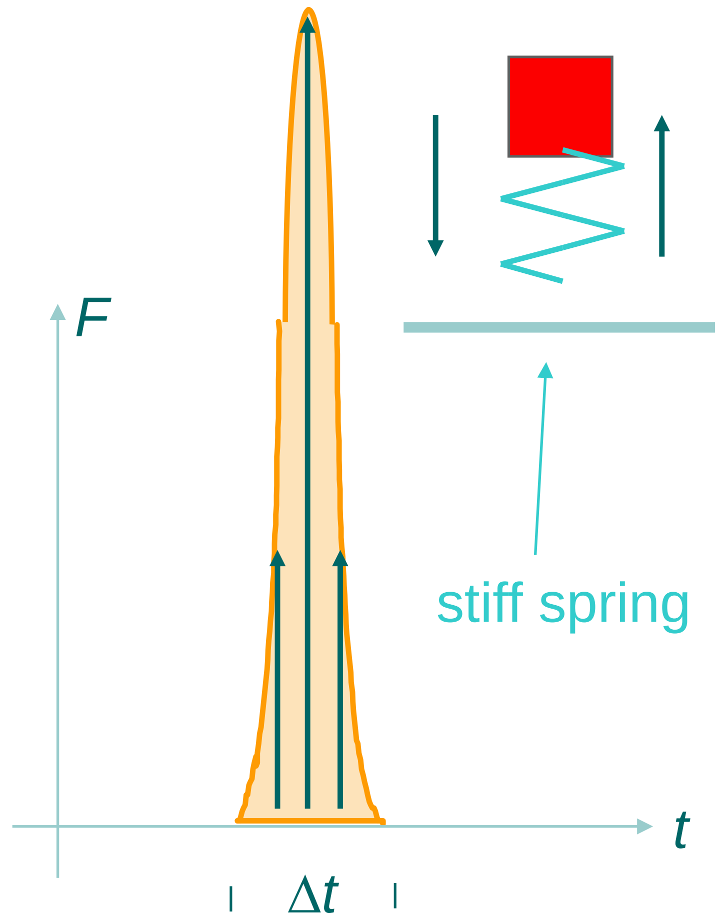


Impulse has units of *Ns*.

Extra: Force and Impulse

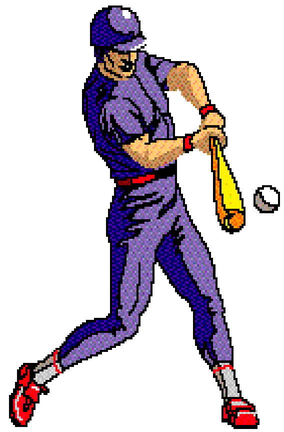


Δt big, ***F*** small



Δt small, ***F*** big

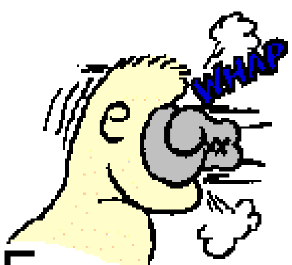
Impulse examples



Follow through increases the time of collision and the impulse

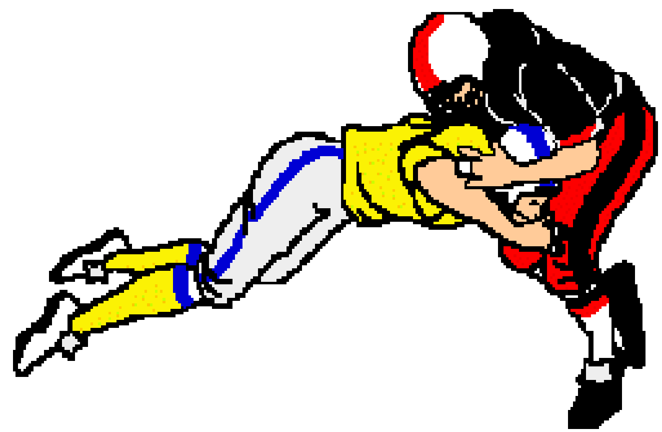
F t = change in momentum

small



F t = change in momentum

large



In football, the defensive player applies a force for a given amount of time to stop the momentum of the offensive player with the ball.

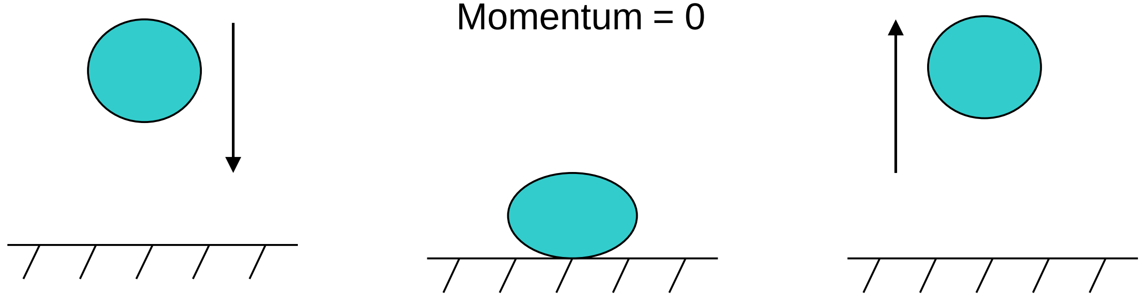
Bouncing

Think about a bouncing ball:

Before it hits the ground:
Speed = v
Momentum = mv

At the moment it hits the ground:
Speed = 0
Momentum = 0

After it leaves the ground:
Speed = v
Momentum = mv



Impulse needed to stop the ball = mv

Impulse needed to accelerate the ball upward = mv

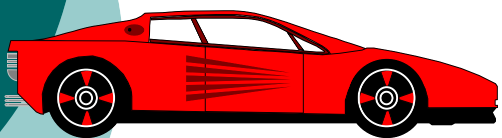
Total Impulse = 2mv

Important point: It only takes an impulse of **mv** to stop the ball.

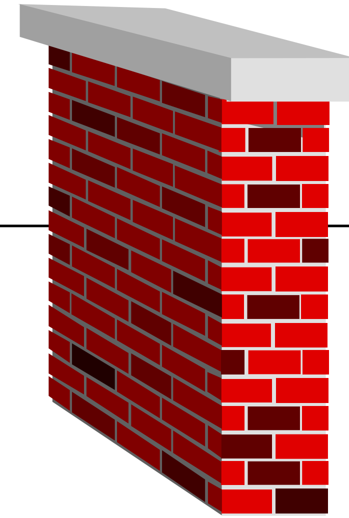
• It takes twice that much (**2mv**) to make it bounce)

•The impulse required to bring something to a stop, in effect “throw it back again” is greater than the impulse required to to merely bring something to a stop

Extra: Stopping Time



$$\Large F\,\small t = \small F\,\Large t$$

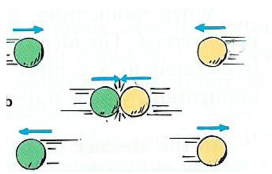


Imagine a car hitting a wall and coming to rest. The force on the car due to the wall is large (big *F*), but that force only acts for a small amount of time (little *t*). Now imagine the same car moving at the same speed but this time hitting a giant haystack and coming to rest. The force on the car is much smaller now (little *F*), but it acts for a much longer time (big *t*). In each case the impulse involved is the same since the change in momentum of the car is the same. Any net force, no matter how small, can bring an object to rest if it has enough time. A pole vaulter can fall from a great height without getting hurt because the mat applies a smaller force over a longer period of time than the ground alone would.

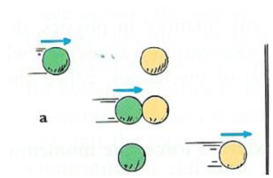
Collisions

Net momentum before collision = net momentum after collision

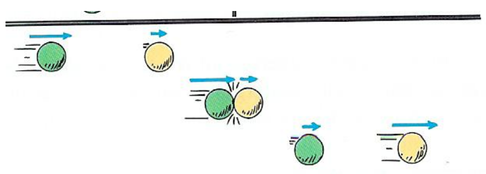
Elastic collisions
- No kinetic energy lost to heat, etc



2 billiard balls collide head on
momentum is the same for each before and after

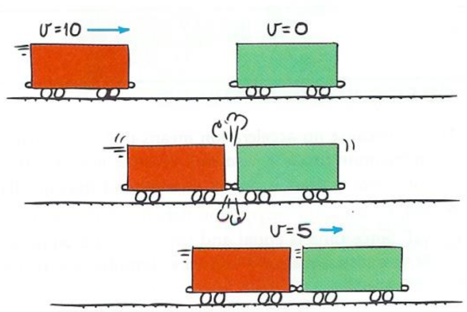


1 billiard balls collide with a stationary one
momentum is the same before and after



2 billiard balls moving in the same direction collide
momentum is the same before and after

Inelastic collisions
- Some kinetic energy lost to heat, etc



Upon collision, the cars stick together
The total mass moves slower, but the momentum of the 2 cars together is the same as the momentum of the system before the collision.

Collisions

- **Elastic collision**
  - A collision in which the total momentum and the total KE are conserved
- **Perfectly inelastic collision**
  - A collision in which two objects stick together after colliding and move together as one mass